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A Fast Scheme for Forward Analysis of Nonlinear Electromagnetic Problems

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Abstract. In this paper, a rapid forward scheme is proposed for simulating the magnetic flux leakage (MFL) signals from a ferromagnetic material. By storing the unflawed magnetic field in databases, the analysis area of the magnetic field perturbation is reduced considering that only magnetic field in the vicinity of the crack can be significantly perturbed. Implemented to the FEM-BEM hybrid code of the polarization method, the new method is validated by comparing the numerical results of the fast forward solver with those of the conventional FEM-BEM code.

1. Introduction

Shape reconstruction of cracks in a structural component of magnetic material from the MFL signals is still a challenging subject to be solved. A difficulty of this inverse problem arises from the numerical simulation of the nonlinear magnetic problem in term of the calculation burden. Recently, numerical method has been developed for simulating both the static and alternating MFL signals on the basis of the FEM-BEM hybrid strategy and the polarization approach [1],[2]. This method, however, still has difficulty to be applied to the inversion of the MFL signals because a fine mesh is required for a good numerical result. To overcome this difficulty, a scheme capable to reduce the analysis region is proposed in this paper based on features of the field perturbation due to a crack [3],[4]. Though the unflawed field data is indispensable in this new scheme, one does not need much CPU time to get them by using a database strategy. Numerical results show that the proposed method can gives excellent prediction of MFL results in a much smaller calculation burden.

2. Basic formulation of the fast forward solver

For a static nonlinear electromagnetic problem, the governing equations can be written as

$$\frac{1}{\mu_0}\nabla^2\mathbf{A} = \nabla\times\mathbf{M} \quad \text{(in material)} \tag{1}$$

$$\frac{1}{\mu_0}\nabla^2\mathbf{A} = \mathbf{J}_0 \quad \text{(in air)} \tag{2}$$

for a material with a crack, and under a constitutive relation of

$$\mathbf{M} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{F}(\mathbf{B}) \cdot \tag{3}$$

In case without a crack, the governing equations are

$$\frac{1}{\mu_0}\nabla^2\mathbf{A}^u = \nabla\times\mathbf{M}^u \quad \text{(in material)} \tag{4}$$

$$\frac{1}{\mu_0}\nabla^2\mathbf{A}^u = \mathbf{J}_0 \quad \text{(in air)} \tag{5}$$

Subtracting (4) and (5) from (1) and (2), one can obtains

$$\frac{1}{\mu_0}\nabla^2\mathbf{A}^f = \nabla\times\mathbf{M}^f \quad \text{(in material)} \tag{6}$$

$$\frac{1}{\mu_0}\nabla^2\mathbf{A}^f = 0 \quad \text{(in air)} \tag{7}$$

where, $\mathbf{A}$ is the magnetic vector potential for material with a crack, $\mathbf{A}^u$ is the magnetic vector potential for the unflawed material, $\mathbf{A}^f = \mathbf{A} - \mathbf{A}^u$ is the perturbation of the two vector potentials, $\mathbf{M}$ is the vector of the magnetization in the material with a crack, $\mathbf{M}^u$ is the magnetization vector for the same material but with the crack absent, $\mathbf{M}^f = \mathbf{M} - \mathbf{M}^u$ is the difference of the two magnetization vectors, and the B-H curve of the material is expressed as $H=F(B)$.

Using the normal FEM-BEM hybrid procedure [1], (6) and (7) become to a system of linear equations as

$$[K]\{A^f\} = [S]\{M^f\}. \tag{8}$$

Where, [K], [S] are coefficient matrices same with those obtained from (1) and (2) for the unflawed material.

Usually, it is not necessary to discretize (6) and (7) on the same analysis region for solving (1) and (2) because the perturbation of the magnetic field is significant only at points in the vicinity of the crack. This means that the computational time can be greatly reduced by solving (6) and (7) on a reduced analysis region. On the other hand, as **B**, **M** are needed in the constitutive relation (3), the magnetization distribution in the material without cracks is indispensable, i.e., a forward analysis on the full analysis region is necessary before to solve (6), (7). However, once this unflawed magnetic field being calculated, it is not necessary to calculate them again for the material with a crack in a varied shape. This feature makes the new algorithm an attractive tool to solve the forward problems for both the model based inverse method and in AI approach.

3. Numerical implementation

In practical, the magnetic field perturbation is obtained with the following procedure:

- Step 0. Establishing the database for $\mathbf{A}^u$, $\mathbf{M}^u$, and $\mathbf{B}^u$.
- Step 1. Calculating $\mathbf{A}^u$, $\mathbf{M}^u$, $\mathbf{B}^u$ at each node of the reduced region from the database above by using interpolation technique.
- Step 2. Setting initial $\mathbf{M}^f$ as $-\mathbf{M}^u$ in crack element and 0 in others.
- Step 3. Solving A^f from (8) and calculating **B** with $\mathbf{B}=\mathbf{B}^f+\mathbf{B}^u$.
- Step 4. Calculating M with B-H curve, and setting $\mathbf{M}^f=-\mathbf{M}^u$ for crack elements and $\mathbf{M}^f=\mathbf{M}-\mathbf{M}^u$ for other elements.
- Step 5. Repeating step 3 and step 4 until convergent.
- Step 6. Calculating MFL signal from $\mathbf{M}^f$.

4. Numerical Results

In the follows, a thick ferromagnetic plate (25 mm in thickness) of A533B steel and a horse-shoe yoke of pure iron are considered as the inspection object and the magnet of the MFL testing. A crack of OD50% depth, 8mm length, and 0.5mm opening is taken as an example for validating the proposed method. Three small plate models with different size are considered as the reduced analysis regions (the element numbers for 3 cases are respectively 1760, 2288 and 2880 while the element number of the full system is used as 10000). In Fig.1, results of the full system and the results using the rapid forward solver with reduced analysis regions are depicted. The CPU times for the 3 cases are respectively 3 minutes, 5 minutes, and about 8 minuets while about 2 hours are needed for the full system. From these results, one can find that even for the smallest region in the 3 cases, the error is below 10%. These results show that the proposed algorithm is valid from the points of view of both the calculation speed and analysis accuracy.

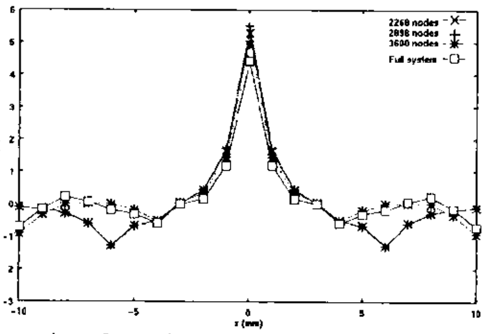

Fig.1 Comparison of numerical results of the fast scheme and the conventional code

5. Conclusions

In this paper, a scheme for fast simulation of the MFL signals is proposed. The numerical results show that the fast forward solver has both a high accuracy and a high speed if the unflawed data being calculated and stored in databases *a priori*. These features make the new algorithm especially feasible in the inverse analysis based on either an optimization method or an artificial intelligent approach.

References

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