# 3D Nonlinear Static Magnetic Field Simulation using an Integral Method

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Abstract. A new method for simulation of nonlinear static magnetic field in three-dimensional structures, including permanent magnets and ferromagnetic materials, is proposed. The nonlinear media are replaced with a linear one, having vacuum permeability and a magnetization iteratively corrected as a function of magnetic flux density. For each derived linear problem, a 3D Green functions formula is used to compute the averaged value of magnetic flux density on subdomains. The convergence of the iterative solution is enhanced using an overrelaxation factor.

## 1. Introduction

Recent developments of new techniques for Nondestructive Testing of thick magnetic structures required a fast and reliable simulator for static magnetic field phenomena. Both computation efficiency and accuracy are expected and, for inverse problem solving, a large amount of successive computations is necessary. All these conditions advocate strongly for an integral method approach, as suggested in [1]. Description of crack regions or regions with different magnetic characteristics implies only modification in the material characteristics used for the iterative correction of magnetization. Hence, Green function coefficients are computed only one for the first problem treated. In the case when mobile active parts are present (the magnetostatic approximation is preserved imposing small scanning velocities), only a small part of the coefficients, requiring also smaller computational effort are recomputed for every position.

## 2. Treatment of the nonlinearity

Let the whole space  $\Omega = \Omega_0 \cup \Omega_m$  be the problem domain,  $\Omega_m$  being the ferromagnetic domain and  $\Omega_0$  the air. For the ferromagnetic media  $\Omega_m$ , the nonlinear magnetic constitutive relationship:

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) \tag{1}$$

is rewritten as  $\mathbf{H} = \hat{F}(\mathbf{B})$ . Maxwell equations in the magnetostatic limits are fulfilled inside the problem domain. The nonlinear magnetic media are replaced with a linear, homogenous one, having vacuum permeability and a magnetization iteratively corrected by a fixed-point procedure [2]. Homogenization is imposed by the integral method used to solve the linear magnetic field problem. This magnetization term is described as:

$$\mathbf{M} = \frac{1}{\mu_0} \mathbf{B} - \hat{\mathbf{F}}(\mathbf{B}) = \hat{\mathbf{G}}(\mathbf{B}), \tag{2}$$

The algorithm for the iterative solution is as follows: a) An arbitrary value  $\mathbf{M}^{(0)}$  is given; b) The magnetic field  $(\mathbf{B}^{(n)},\mathbf{H}^{(n)})$  is computed;

- c)  $\mathbf{M}^{(n)}$  is corrected with relation (2):  $\mathbf{M}^{(n)} = \hat{\mathbf{G}}(\mathbf{B}^{(n)})$ ;

Step b) and c) are repeated until an imposed error  $\|\mathbf{M}^{(n)} - \mathbf{M}^{(n-1)}\|_{L^2(\Omega)}$  is achieved.

The main drawback of the polarization method is its slow, because linear, convergence speed. An important improvement in terms of computation speed was obtained by overrelaxation. This numerical procedure was described extensively elsewhere [2].

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#### 3. Computation of magnetic field

The sources of magnetic field are the current sources, the permanent magnets and the magnetization in ferromagnetic parts. Only active parts (permanent magnets and ferromagnetic materials) are meshed. On each hexahedral element the averaged value on subdomain for magnetic flux density is computed as:

$$\tilde{\mathbf{B}}_{i} = \frac{1}{v_{i}} \oint_{\partial \omega_{i}} (\mathbf{n} \times \mathbf{A}) dS_{i} \qquad (3)$$

where  $v_i$  is the volume of the hexahedral element  $\omega_i$ , A is the magnetic vector potential, n is the normal on the subdomain external surface. The magnetic vector potential is computed using Biot-Savart law and is expressed as:

$$A = A_m + A_0 \quad , \tag{4}$$

where  $A_m$  is the value due to the magnetization and  $A_0$  is the value due to the current sources:

$$\mathbf{A}_{m} = \frac{\mu_{0}}{4\pi} \int_{\Omega} \frac{\mathbf{M} \times \mathbf{n}}{R} dv \,, \tag{5}$$

$$\mathbf{A}_0 = \frac{\mu_0}{4\pi} \int_{\Omega} \frac{\mathbf{J}_0}{R} dv \tag{6}$$

Equation (3) is then transformed as:

$$\tilde{\boldsymbol{B}}_{i} = \tilde{\boldsymbol{B}}_{m_{i}} + \tilde{\boldsymbol{B}}_{0_{i}} \quad , \tag{7}$$

where each term is expressed as:

$$\widetilde{\mathbf{B}}_{m_i} = -\frac{1}{v_i} \sum_{k=1}^{n_F} \overline{\widetilde{\gamma}}_{ik} \mathbf{M}_k \,, \tag{8}$$

$$\widetilde{\mathbf{B}}_{0i} = \frac{1}{\nu_i} \oint_{\partial \omega_i} (\mathbf{n} \times \mathbf{A}_0) dS_i \qquad (9)$$

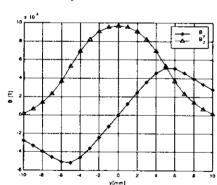
The coefficients in (9) are computed as:

$$\bar{\bar{\gamma}}_{ik} = \frac{\mu_0}{4\pi} \oint_{\partial \omega_i} \oint_{\partial \omega_k} \frac{\mathbf{n}_k \cdot \mathbf{n}_i - (\mathbf{n}_k \cdot \mathbf{n}_i)}{R} dS_k dS_i$$
 (10)

where ";" stands for the dyadic product.

### 4. Results and conclusions

A plate with dimensions 50 mm × 50 mm × 10 mm is energized using a cylindrical coil carrying total current 1 AT. The plate material is made from nonlinear ferrite F82H [3]. The lift-off of the coil is 1 mm. The



dimensions of the coil are: height 2 mm, inner radius 2 mm, outer radius 4 mm. Figure 1 shows the distribution of the magnetic field on a scan line oriented along y-axis, with lift-off 0.5 mm, over the plate. The results were obtained after 11 iterations

A novel method for computing the nonlinear magnetic static field problem was proposed. The Green function coefficients for the threedimensional problem are computed only once and an iterative correction of the magnetization term is applied. The convergence of the method is theoretically assured and may be accelerated by using an overrelaxation factor.

Fig. 1. Field measured over the plate

## References

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