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Sensitivities for a Synchronous Generator

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Abstract: The paper presents a very useful method for computing the sensitivities of electrical machines, taking into account the non-linearity of the ferromagnetic bodies. Firstly, the magnetic field in non-linear media and the performances of the given machine are calculated. The non-linear medium is replaced with a linear medium having a magnetic polarization that is iteratively corrected by the flux density. Each linear field problem is solved using the periodic Green function. Secondly, the difference of the magnetic field due to the small modification is computed directly. The polarization of the eliminated ferromagnetic domain is the field source. The **B** - **I** relation is given by Frechet first derivative.

1. Introduction

Several codes have been developed in the last years for the computation of electromagnetic fields. These codes use mainly Finite Element (FEM), Finite Difference (FDM) or the Boundary Element (BEM) methods. Airgaps of electrical machines beeing very small compared with other parts of the machine, the mesh used in FEM and FDM must be adapted to these structures. A great number of unknowns will result in order to obtain a sufficient fine mesh in the airgap zone. Nonlinearity of magnetic parts and the geometry of the airgap region, rather complicated, lead to major difficulties in the use of BEM.

The aim of this paper is to report a very efficient approach to the field computation, based on the evaluation of the difference of two fields due to small geometrical modifications. The first field corresponds to the given machine, while the second one, to the modified machine. For small modifications, these two solutions are almost the same. Consequently they must be very accurately calculated in order to obtain a suitable difference. In this paper we propose a procedure for direct computation of the difference of these two magnetic fields, due to small, arbitrary modifications. The results are useful for optimisation of the machine. This approach is outlined in [1].

2. Solving the Non-linear Problem

In domain Ω the magnetic field verifies equations: $\nabla \times \mathbf{H} = \mathbf{J}$, $\nabla \mathbf{B} = 0$, $\mathbf{H} = \hat{F}(\mathbf{B})$. The iterative polarization method is used [2]. The constitutive relation **B-H** is replaced by: $\mathbf{B} = \mu \mathbf{H} + \mathbf{I}$. The nonlinearity is contained in the polarization **I**: $\mathbf{I} = \mathbf{B} - \mu \hat{F}(\mathbf{B}) = \hat{G}(\mathbf{B})$. For any **I** we have one and only one magnetic field (**B**,**H**) [3]. The function $\mathbf{I} \rightarrow \mathbf{B} = \hat{B}(\mathbf{I})$ is nonexpansive. If we choose $\mu < 2\,\mu_{min}$ then the function $\hat{G}$ is a contraction. Generally $\mu_{min} \geq \mu_0$ and therefore we can replace the nonlinear medium by a linear one having the permeability of the vacuum. The contraction factor θ is: $\theta = 1 - \frac{\mu_0}{\mu_{max}}$. For a given polarization **I**, the magnetic field in a linear media with permeability μ is computed. After, the polarization is corrected by function $\hat{G}$. The convergence of the above iterative procedure may be improved using the overrelaxation described in [2].

3. The Linear Field Problem

We consider that Ω is the whole space $\mathbf{R}^2$. We replace all ferromagnetic media by a medium having a vacuum permeability μ_0. The vector potential $\mathbf{k}(A_0 + A)$ is used. The current density is constant on subdomains and its vector potential is:

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$$A_0 = -\frac{\mu_0}{2\pi}\sum_{k=1}^{n_J} J_k \int_{\gamma_k} \ln R\, dS = \frac{\mu_0}{4\pi}\sum_{k=1}^{n_J} J_k \oint_{\partial\gamma_k} \mathbf{nR}\left(\ln R - \frac{1}{2}\right) dl = \sum_{k=1}^{n_J} a_k J_k \qquad (1)$$

The ferromagnetic regions are divided in n_f subdomains ω_k where the polarization $\mathbf{I}$ is constant. The magnetic vector potential and the flux density, computed by $\mathbf{B}_T = \mathbf{B}_0 + \tilde{\mathbf{B}}$ where $\mathbf{B}_0$ is given by current density, are:

$$A = \frac{1}{2\pi}\sum_{k=1}^{n_f} \mathbf{I}_k \oint_{\partial\omega_k} \ln R\, d\mathbf{l} = \sum_{k=1}^{n_f} \mathbf{z}_k \mathbf{I}_k \qquad \mathbf{B}_0 = -\frac{\mu_0}{2\pi}\sum_{k=1}^{n_J} J_k \oint_{\partial\gamma_k} \ln R\, d\mathbf{l} = \sum_{k=1}^{n_J} \mathbf{u}_k J_k . \qquad (2)$$

and $\tilde{\mathbf{B}}$ is the average value of the flux density given by polarization:

$$\tilde{\mathbf{B}}_i = \frac{1}{\sigma(\omega_i)}\int_{\omega_i} \mathbf{B}\, dS = \frac{-1}{2\pi\sigma(\omega_i)}\sum_{k=1}^{n_f} \oint_{\partial\omega_k}\oint_{\partial\omega_i} \ln R(\mathbf{I}_k\, d\mathbf{l}_k)\, d\mathbf{l}_i = -\frac{1}{\sigma(\omega_i)}\sum_{k=1}^{n_f} \bar{\bar{\alpha}}_{ik}\mathbf{I}_k \qquad (3)$$

where $\quad \bar{\bar{\alpha}}_{ik} = \frac{1}{2\pi}\oint_{\partial\omega_i}\oint_{\partial\omega_k} \ln R\, d\mathbf{l}_i ; d\mathbf{l}_k$ and ";" is the dyadic product.

4. The Difference of the Magnetic Fields

By computing the magnetic field $(\mathbf{B}_0, \mathbf{H}_0)$ in the given machine we obtain the polarization $\mathbf{I}_0$ in all ferromagnetic subdomains. The small modifications of the geometry seem to remove certain ferromagnetic subdomains. The difference $(\Delta\mathbf{B}, \Delta\mathbf{H})$ of the magnetic fields verifies [3] relations $\nabla \times \Delta\mathbf{H} = 0$, $\nabla\Delta\mathbf{B} = 0$ and constitutive relation

$$\Delta\mathbf{B} = \begin{cases} \mu_0\,\Delta\mathbf{H} - \mathbf{I}_0, & \text{in removed ferromagnetic domains;} \\ \mu_0\,\Delta\mathbf{H} + \Delta\mathbf{I}, \text{ with } \Delta\mathbf{I} = \left.\frac{d\hat{G}}{d\mathbf{B}}\right|_{\mathbf{B}_0}(\Delta\mathbf{B}), & \text{in remained ferromagnetic domains;} \\ \mu_0\Delta\,\mathbf{H}, & \text{in air.} \end{cases} \qquad (4)$$

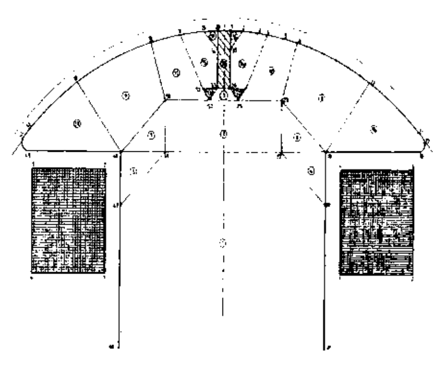

Fig.1. Rotor pole

The Frechet derivative $\frac{d\hat{G}}{d\mathbf{B}}$ results from Frechet derivative of the local B-H constitutive relation. For example, for isotropic media we have:

$$\frac{d\mathbf{g}}{d\mathbf{B}} = \frac{1}{B^2}\frac{dg}{d\mathbf{B}}(\mathbf{B};\mathbf{B}) + \frac{g(B)}{B}\mathbf{1} - \frac{g(B)}{B^3}(\mathbf{B};\mathbf{B})$$

5. Numerical Results

A rotor pole of a small synchronous generator is presented in the Fig. 1. A slot will be made in the middle of the rotor pole (see the hatch in Fig.1). In Table 1 the dependence of the magnetic flux of a stator phase as function of stator - rotor angle is described.

Table 1 Stator magnetic fluxes for a small synchronous generator

Angle	0	15	30	45	60	75	90
With slot	.57E-7	-.02478	-.0473	-.06516	-.07828	-.08559	-.08831
Without slot	.65E-7	-.02613	-.0495	-.06822	-.08151	-.08891	-.09167
Difference	-.2E-7	.00133	.0021	.00302	.00317	.00327	.00331

6. References

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