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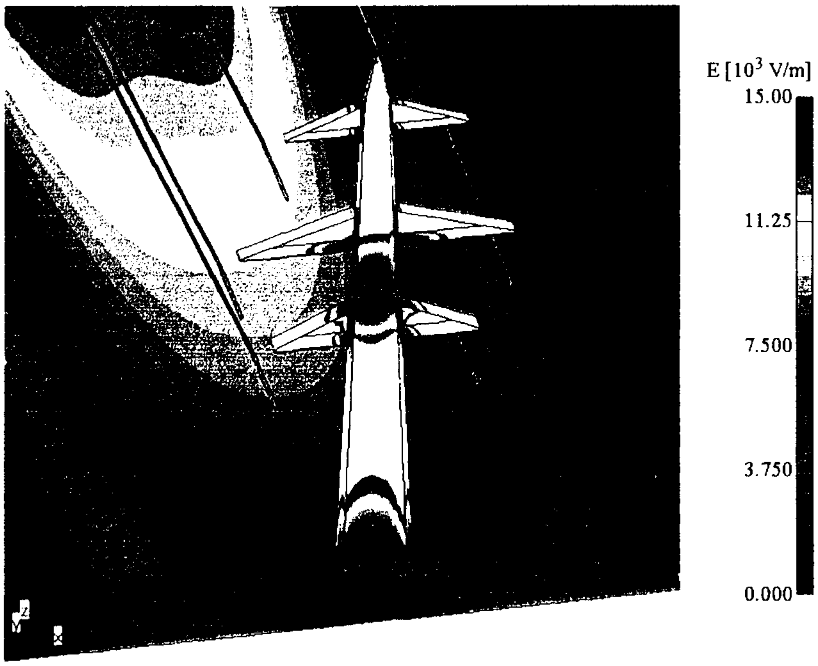

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Integral Formulation for Eddy Current and Forces Computation in Moving Bodies

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***Abstract*-This paper presents an integral formulation for the computation of eddy current and forces in rigid moving conductors, in the presence of magnetic media. The quasistationary Maxwell equations are written in local reference frames associated with moving bodies. Tree-cotree decomposition, Galerkin procedure, time stepping for time integration and Picard-Banach iterations for solving the non-linear problem are used. The forces are computed using the Maxwell stress tensor integrated over a surface including each body.**

I. INTRODUCTION

The eddy current analysis for deformable bodies, including the force computation was performed in [1] in a differential formulation. The use of an FEM-BEM method, including the coupled mechanical and electromagnetic problem is shown in [2]. In [3], a hybrid method is used for eddy currents computation in non-deformable moving bodies, including non-linear media but the velocity is imposed. In both [2] and [3] a Lagrangian formulation is employed. In a previous paper [4], an integral formulation, in terms of an two components electric vector potential is used for the eddy currents computation in non-linear media.

In this paper, the method proposed in [4] is developed by computing the forces using the Maxwell stress tensor integration.

II. PROBLEM FORMULATION

The model is based on the magnetoquasistatic limit of Maxwell equations and the following constitutive relationships:

$$\mathbf{E} = \rho \mathbf{J} \quad \text{in } \Omega_c, \tag{1}$$

$$\mathbf{H} = f(\mathbf{B}) \ \text{ in } \Omega_f, \tag{2}$$

where $\mathbf{J}$ is the current density, $\mathbf{E}$ is the electric field, ρ stands for the resistivity, Ω_c is the conducting domain and Ω_f is the ferromagnetic domain. The domain of the sources is Ω_0.

Constitutive equation (2) is equivalent to:

$$\mathbf{B} = \mu_0 (\mathbf{H}+\mathbf{M}) \tag{3}$$

$$\mathbf{M} = g(\mathbf{B}) = \nu_0 \mathbf{B} - f(\mathbf{B}) \qquad (4$$

where μ_0 is the vacuum permeability and $\nu_0=1/\mu_0$. We assume f to be uniformly monotone and verify Lipschitz condition.

We consider a set of N conducting and/or ferromagnetic bodies B_k, each of them moving with a rigid velocity $\mathbf{v}_k$. In the local frame of any moving body the electric field is given by:

$$\mathbf{E}(\mathbf{x},\tau) = -\frac{\partial \mathbf{A}(\mathbf{x},t)}{\partial t} - \nabla W \tag{5}$$

where W is the scalar electric potential and $\mathbf{A}$ is the divergence-free magnetic vector potential which can be split into three terms due to the imposed currents, to the induced eddy currents and to the magnetization [4]:

$$\mathbf{A} = \frac{\mu_0}{4\pi}\int_{\Omega_c} \frac{\mathbf{J}}{r} dv + \frac{\mu_0}{4\pi}\int_{\Omega_f} \frac{\nabla\times\mathbf{M}}{r} dv + \mathbf{A}_0$$

and

$$\mathbf{A}_0 = \frac{\mu_0}{4\pi}\int_{\Omega_0} \frac{\mathbf{J}_0}{r} dv \cdot$$

The source domains could be either massive coils or filamentary wiring.
The integral equation in terms of $\mathbf{J}$ obtained is:

$$\begin{aligned} &\rho\mathbf{J} + \frac{\mu_0}{4\pi}\frac{d}{dt}\int_{\Omega_c} \frac{\mathbf{J}}{r} dv + \nabla W = \\ &-\frac{\mu_0}{4\pi}\frac{d}{dt}\int_{\Omega_0} \frac{\mathbf{J}_0}{r} dv - \frac{\mu_0}{4\pi}\frac{d}{dt}\int_{\Omega_F} \frac{\nabla\times\mathbf{M}}{r} dv \end{aligned} \tag{6}$$

The force applied to each body is computed using the integration of the Maxwell stress tensor over a surface including the body:

$$\mathbf{F} = \frac{1}{\mu_0}\oint_{\Sigma}\left[(\mathbf{nB})\mathbf{B} - \mathbf{n}\frac{B^2}{2}\right] dA \tag{7}$$

where $\mathbf{B}$ is due to the eddy-currents induced in the conducting domains Ω_c, to the magnetization in Ω_f and to the imposed currents in massive or filamentary coils:

$$\begin{aligned} \mathbf{B} = &\frac{\mu_0}{4\pi}\int_{\Omega_c} \frac{\mathbf{J}\times\mathbf{R}}{R^3} dv + \frac{\mu_0}{4\pi}\int_{\Omega_F} \frac{(\nabla\times\mathbf{M})\times\mathbf{R}}{R^3} dv + \\ &+\frac{\mu_0}{4\pi}\int_{\Omega_0} \frac{\mathbf{J}_0\times\mathbf{R}}{R^3} dv + \sum_{k=1}^{n} \frac{\mu_0 i_k}{4\pi}\int_{\Gamma_k} \frac{d\mathbf{l}\times\mathbf{R}}{R^3} dv \end{aligned} \tag{8}$$

We solve then the mechanical equation for obtaining the velocity and the displacement of each body.

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III. Numerical Approach

The current density $\mathbf{J}$ is expressed in terms of cotree edge shape functions $\mathbf{T_k}$ whereas the magnetization $\mathbf{M}$ is approximated as a piecewise uniform field. Using Galerkin approach, we obtains the system of equations:

$$[R]\,[\alpha]+\frac{d}{dt}[L][a]=[U]+[V]\,, \tag{9}$$

where $[R]$, $[L]$, $[U]$, $[V]$ and $[\alpha\ ]$ are specified in previous paper [4].

For solving this system, time integration for the time stepping and Picard-Banach iterations for the nonlinear treatment are used. Motion is taken into account in (9) in the source terms $[U]$, $[V]$ as well as by updating parts of matrix $[L]$ at each time step.

The magnetization is iteratively corrected at each step of time using the relation (4). Overrelaxation methods may be used in order to accelerate convergence [5]. The flux density $\mathbf{B}$ is calculated as an average value in each ferromagnetic element and could be expressed as:

$$\tilde{\mathbf{B}}_i=\frac{1}{v_i}\sum_{k=1}^{n_c}\boldsymbol{\beta}_{ik}\times\mathbf{J}_k-\frac{1}{v_i}\sum_{p=1}^{n_f}\bar{\bar{\gamma}}_{ip}\mathbf{M}_p+\mathbf{B}_0 \tag{10}$$

where:

$$\boldsymbol{\beta}_{ik}=\frac{\mu_0}{4\pi}\oint_{\partial\omega_i}\int_{\omega_k}\frac{\mathbf{n}_i}{r}dv_k\,dS_i\,, \tag{11}$$

$$\bar{\bar{\gamma}}_{ip}=\frac{\mu_0}{4\pi}\oint_{\partial\omega_i}\oint_{\partial\omega_p}\frac{\mathbf{n}_p;\mathbf{n}_i-(\mathbf{n}_p\mathbf{n}_i)}{r}dS_p\,dS_i\,. \tag{12}$$

n_c is the number of conducting tetrahedral sub-domains, n_f is the number of hexahedral sub-domains and $\mathbf{B}_0$ is the imposed magnetic flux density.

The magnetic flux density $\mathbf{B_J}$ due to the eddy-currents in the tetrahedral sub-domains of the conducting domain is:

$$\mathbf{B}_J=\frac{\mu_0}{4\pi}\sum_{k=1}^{n_c}\mathbf{J}_k\times\int_{\omega_k}\frac{\mathbf{R}}{R^3}dv=\frac{\mu_0}{4\pi}\sum_{k=1}^{n_c}\mathbf{J}_k\times\int_{\partial\omega_k}\frac{\mathbf{R}}{R}dA \tag{13}$$

where $\mathbf{J_k}$ is the current density in tetrahedral sub-domain k of Ω_c(see Figure 1).

The component of the magnetic flux density due to the magnetization is $\mathbf{B_M}$:

$$\begin{aligned}\mathbf{B}_M&=\frac{\mu_0}{4\pi}\sum_{k=1}^{n_f}\int_{\omega_f}\frac{-(\mathbf{n}\times\mathbf{M}_k)\times\mathbf{R}}{R^3}dv=\\&\frac{\mu_0}{4\pi}\sum_{k=1}^{n_f}\int_{\omega_f}\frac{(\mathbf{nR})\mathbf{M}_k-(\mathbf{M}_k\mathbf{R})\mathbf{n}}{R^3}dv=\\&\frac{\mu_0}{4\pi}\sum_{k=1}^{n_f}\mathbf{M}_k\cdot\int_{\omega_f}\frac{(\mathbf{nR})-(\mathbf{n};\mathbf{R})}{R^3}dv\end{aligned} \tag{14}$$

Here, $\mathbf{M_k}$ is the magnetization in hexahedral sub-domain k of Ω_f (see Figure 1).

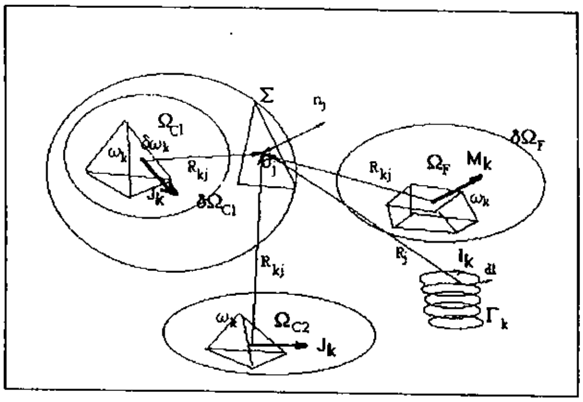

Fig. 1 Computation of the magnetic flux density on the Maxwell stress tensor integration surface

For sake of simplicity, we could use as integration surface for the Maxwell stress tensor one surface derived from the exterior surface of one body using a spatial extrapolation. Using relations (13) and (14), relation (8) become:

$$\begin{aligned}\mathbf{F}=\frac{\mu_0}{16\pi^2}\sum_{j=1}^{n_s}\int_{\sigma_j}\Bigg[&\left(\mathbf{n}_j\sum_{k=1}^{n_c}\mathbf{J}_k\times\mathbf{N}_{kj}+\mathbf{n}_j\sum_{k=1}^{n_f}\mathbf{M}_k\cdot\bar{\bar{\mathbf{Q}}}_{kj}+\mathbf{n}_j\mathbf{S}_j\right)-\\&\left(\sum_{k=1}^{n_c}\mathbf{J}_k\times\mathbf{N}_{kj}+\sum_{k=1}^{n_f}\mathbf{M}_k\cdot\bar{\bar{\mathbf{Q}}}_{kj}+\mathbf{S}_j\right)\\&\frac{\mathbf{n}_j}{2}\left(\sum_{k=1}^{n_c}\mathbf{J}_k\times\mathbf{N}_{kj}+\sum_{k=1}^{n_f}\mathbf{M}_k\cdot\bar{\bar{\mathbf{Q}}}_{kj}+\mathbf{S}_j\right)^2\Bigg]dA_j\end{aligned} \tag{15}$$

where:

$$\mathbf{N}_{kj}=\int_{\partial\omega_k}\frac{\mathbf{R}_j}{R_j}dA \tag{16}$$

$$\bar{\bar{\mathbf{Q}}}_{kj}=\int_{\omega_f}\frac{(\mathbf{nR}_j)-(\mathbf{n};\mathbf{R}_j)}{R^3}dv \tag{17}$$

$$\mathbf{S}_j=\sum_{k=1}^{n}i_k\int_{\Gamma_k}\frac{\mathbf{dl}\times\mathbf{R}_j}{R_j^{\,3}}+\int_{\Omega_0}\frac{\mathbf{J}_0\times\mathbf{R}}{R^3} \tag{18}$$

For compute those terms Gauss integration and analytical formulas are used.

IV. Results

The problem of a moving coil over a thin, conducting plate with the dimensions used by Kameari in [6] is used here for validating the linear solver. We have used a fine mesh, with

2975 tetrahedrons and 1239 active edges. The time step is correlated with the velocity and the dimension of the mesh in the movement direction as the Courant number C < 1. In Figure 2 is shown the eddy-current distribution in the rectangular plate with a speed of 500 km/h (138.9 m/s). The total power losses vs. the velocities of the coil compared with the analytical result are shown in Figure 2 for five different velocities (v = 100 km/h, 200 km/h, 300 km/h, 400 km/h and 500 km/h).

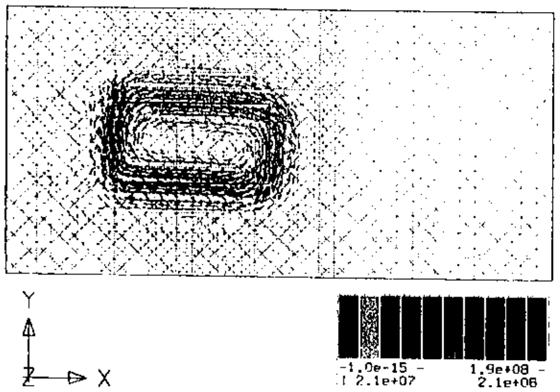

Fig. 2 Eddy-current distribution in the rectangular plate with a speed of 500 km/h (138.9 m/s), computed using a fine mesh with 2975 tetrahedrons and 1239 active edges

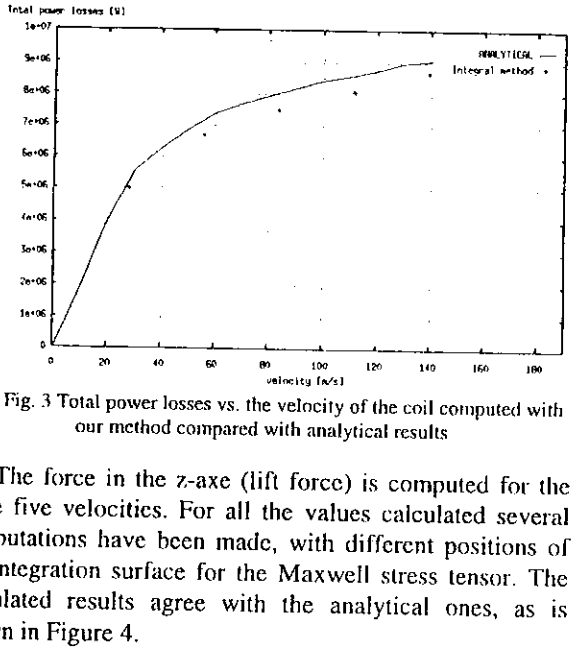

Fig. 3 Total power losses vs. the velocity of the coil computed with our method compared with analytical results

The force in the z-axe (lift force) is computed for the same five velocities. For all the values calculated several computations have been made, with different positions of the integration surface for the Maxwell stress tensor. The calculated results agree with the analytical ones, as is shown in Figure 4.

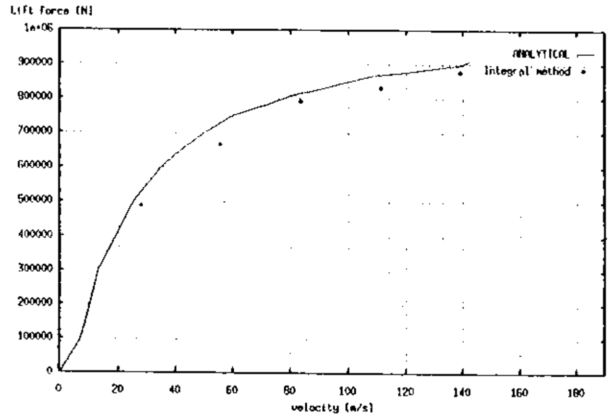

Fig. 4 Force in the z-axe direction (lift force) for the velocities v = 100 km/h , 200 km/h, 300 km/h, 400 km/h and 500 km/h compared with the analytical values

V. Conclusion

A method for computation of non-linear eddy-currents and forces in moving media was presented. The source domains are both massive coils and filamentary wiring. A tree-cotree decomposition, Galerkin method and an iterative procedure for the treatment of the non-linearity are used. Some results are provided only for conducting media, including the force computation.

VI. Acknowledgment

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References

[1] A. Bossavit, Differential forms and the computation of fields and forces in electromagnetism, *Eur. J. Mech.*, B: Fluids, **10**, no 5, 474-488 (1991)

[2] S. Kurz, J. Fetzer, G. Lehrer, W. Rucker, "A Novel Formulation for 3D Eddy Current Problems with Moving Bodies Using a Lagrangian description and BEM-FEM Coupling", Proc. of COMPUMAG '97, *3-6 Nov 1997, Rio de Janeiro, Brazil, pp. 417-418.*

[3] T. Tarhasaari, A. Koski, K. Forsman, L. Kettunen, "Formulation of the Eddy Current Problem with Hybrid Methods", Proc. of COMPUMAG '97, *3-6 Nov 1997, Rio de Janeiro, Brazil, pp. 787-788.*

[4] R. Albanese, F.I. Hantila, G. Preda, G. Rubinacci, "A Nonlinear Eddy-Current Integral Formulation for Moving Bodies", Proc. of COMPUMAG '97, *3-6 Nov 1997, Rio de Janeiro, Brazil, pp.* 213-214.

[5] M. Chiampi, C. Ragusa, M. Repetto, "Strategies for Accelerating Convergence in Nonlinear Fixed Point Method Solution", Proc. of the 7th IGTE Symposium, *Sep 1996, Graz, Austria,, pp.2 45-250.*

[6] S. Niikura. and A. Kameari, "Analysis of Eddy Current and Force in Conductors with Motion", IEEE Trans. on Magn., Vol. 28, No. 2, pp. 1450-1453, 1992.