Eddy Current Levitation in Ferromagnetic Structures

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The efficiency of electromagnetic levitation devices can be improved by shortening the path of the magnetic flux which can be done by employing a magnetic circuit. The eddy currents are computed using an integral equation, and the ferromagnetic materials nonlinearity is treated by applying the polarization fixed point iterative method, with a dynamic overrelaxation technique. To substantially reduce the computation time, the eddy current integral equation is solved in the frequency domain, with supplementary terms added to account for the motion, and in the equation of motion an average magnetic force over a period is considered.

***Index Terms*— Eddy currents, ferromagnetics, levitation.**

I. Introduction

ENHANCED electromagnetic forces can be obtained in levitating systems, for the same imposed currents, by including magnetic parts that concentrate the magnetic flux to the levitated conductor. The simulation effort for such systems increases spectacularly owing to the additional steps required by the iterative process used for treating the nonlinearity. In this paper, a fast procedure for computing levitated body trajectory in a 2-D setup is proposed.

The numerical solution of the motion equation is obtained by computing the electromagnetic force, position, and velocity at each time step. Using the finite element method (FEM) for computing the electromagnetic field and forces requires mesh deformation or a complicated coupling technique of the sliding meshes associated with the moving parts. By adopting FEM–boundary element method techniques [1], this drawback can be eliminated.

In this paper, the current density integral equation [2], [3], is adopted. Nonlinear treatment is done using the polarization fixed-point method (PFPM) [4]–[6]. This method replaces the nonlinear medium with a linear one, having the vacuum permeability and a magnetic polarization that is iteratively corrected as a function of the magnetic flux density. Thus, the current density integral equation is used, having an additional right-hand side term due to polarization. Only the fundamental component of the magnetic force harmonics spectrum is shown to be important for the motion analysis. Convergence speed of PFPM increases spectacularly if the dynamic overrelaxation procedure proposed in [7] is applied. The mechanical time constant is much greater than the electrical one, in levitation systems. This allows using the average value of force per one electrical period in the numerical solution of the motion equation. Thus, the mechanical time step is correlated with the period of the imposed current.

II. Nonlinearity Treatment

The nonlinear $\boldsymbol{H} = \mathbf{F}(\boldsymbol{B})$ characteristic is treated using PFPM [4]–[6].

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This characteristic is replaced by $\boldsymbol{B} = \mu\boldsymbol{H} + \boldsymbol{I}$ where μ is a constant and $\boldsymbol{I}$ is a nonlinear function of $\boldsymbol{B}$

$$\boldsymbol{I} = \boldsymbol{B} - \mu\mathbf{F}(\boldsymbol{B}) \equiv \mathbf{G}(\boldsymbol{B}). \tag{1}$$

At iteration k, with the known value $\boldsymbol{I}^{(k-1)}$ the magnetic flux density $\boldsymbol{B}^{(k)}$ is computed. Next, $\boldsymbol{I}^{(k)} = \mathbf{G}(\boldsymbol{B}^{(k)})$ is evaluated. When the norm $\|\boldsymbol{I}^{(k)} - \boldsymbol{I}^{(k-1)}\|$ is small enough, the iterations are stopped. The norm is given by $\|\boldsymbol{U}\| = (\frac{1}{T}\int_0^T\int_{\Omega_f}\boldsymbol{U}\cdot\boldsymbol{U}d\Omega\, dt)^{1/2}$, with T being the electrical period and Ω_f the domain occupied by the ferromagnetic bodies. Because the electromagnetic field computation uses the current density integral equation, μ is chosen to be the free space permeability μ_0.

III. Eddy Current Integral Equation

In the reference frame of the coils, Faraday's law for a 2-D setup is

$$\rho J = -\frac{\partial A}{\partial t} + \boldsymbol{k}\cdot(\boldsymbol{v}\times\boldsymbol{B}) + C \tag{2}$$

where ρ is the solid conductor resistivity, $\boldsymbol{k}$ is the Oz axis unit vector, and $\boldsymbol{v}$ is the velocity of the conductor. The magnetic vector potential is $A = A^J + A^{J_i} + A^I$ where A^J, A^{J_i}, A^I are produced by the induced current density J, the imposed current density J_i, and the magnetic polarization $\boldsymbol{I}$, respectively. The magnetic flux density $\boldsymbol{B} = \boldsymbol{B}^{J_i} + \boldsymbol{B}^I$ in (2) is given only by the imposed current density J_i and the polarization $\boldsymbol{I}$.

The eddy current integral equation is (the Appendix)

$$\begin{aligned}\rho J + \gamma\int_\Omega \frac{\partial J(\boldsymbol{r}',t)}{\partial t}\ln\frac{1}{R}ds' &= -\gamma\int_{\Omega_i}\frac{\partial J_i(\boldsymbol{r}',t)}{\partial t}\ln\frac{1}{R}ds' + \gamma\int_{\Omega_i}\left(\boldsymbol{v}\cdot\frac{\boldsymbol{R}}{R^2}\right)J_i(\boldsymbol{r}',t)ds' \\ &\quad - \frac{1}{2\pi}\boldsymbol{k}\cdot\int_{\Omega_f}\frac{\partial\left(\nabla'\times\boldsymbol{I}(\boldsymbol{r}',t)\right)}{\partial t}\ln\frac{1}{R}ds' \\ &\quad + \frac{1}{2\pi}\boldsymbol{k}\cdot\int_{\Omega_f}(\boldsymbol{v}\cdot\frac{\boldsymbol{R}}{R^2})\left(\nabla'\times\boldsymbol{I}(\boldsymbol{r}',t)\right)ds'\end{aligned} \tag{3}$$

where $\gamma \equiv (\mu_0/2\pi)$, $\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}'$, $R = |\boldsymbol{R}|$, with $\boldsymbol{r}$ and $\boldsymbol{r}'$ being the position vectors of the observation and of the source point, respectively. The domains Ω, Ω_i, and Ω_f contain the solid conductors, the coils, and the ferromagnetic bodies, respectively.

IV. Periodic Steady-State Electromagnetic Field

The Fourier series expansion of the periodic magnetic polarization is

$$\boldsymbol{I}(t) = \sum_{n=1,3,\ldots} \left(\boldsymbol{I}_n^{\mathrm{re}}\sqrt{2}\sin(n\omega t) + \boldsymbol{I}_n^{\mathrm{im}}\sqrt{2}\cos(n\omega t)\right). \tag{4}$$

For the numerical computation, only a finite number N of harmonics are retained, $\boldsymbol{I} \cong \boldsymbol{I}_a \equiv \mathbf{Y}(\boldsymbol{I})$. For each harmonic n of the magnetic polarization $\boldsymbol{I}(t)$, the complex representation $\underline{\boldsymbol{I}} = \boldsymbol{I}^{\mathrm{re}} + j\,\boldsymbol{I}^{\mathrm{im}}$ is used and the electromagnetic field problem is solved by employing a phasor representation of (3)

$$\begin{aligned}
&\rho \underline{J}(\boldsymbol{r}) + j\lambda \int_{\Omega} \underline{J}(\boldsymbol{r}') \ln\frac{1}{R} ds' \\
&= -j\lambda \int_{\Omega_i} \underline{J}_i(\boldsymbol{r}') \ln\frac{1}{R} ds' + \gamma \int_{\Omega_i} \left(\boldsymbol{v}\cdot\frac{\boldsymbol{R}}{R^2}\right)\underline{J}_i(\boldsymbol{r}') ds' \\
&\quad -jnf\boldsymbol{k}\cdot\int_{\Omega_f} \nabla'\times\underline{\boldsymbol{I}}(\boldsymbol{r}') \ln\frac{1}{R} ds' + \frac{1}{2\pi}\boldsymbol{k} \\
&\quad \cdot \int_{\Omega_f} \left(\boldsymbol{v}\cdot\frac{\boldsymbol{R}}{R^2}\right)\left(\nabla'\times\underline{\boldsymbol{I}}(\boldsymbol{r}')\right) ds'
\end{aligned} \tag{5}$$

where $\lambda \equiv 2\pi\, nf\gamma$, f is the frequency and $\underline{J} = J^{\mathrm{re}} + j\;J^{\mathrm{im}}$ is the phasor representation of the current density, with $j^2 = -1$.

To simplify the presentation, the index n of the harmonic will be omitted, when it is possible, and the complex quantities will be understood as phasor representations.

For each harmonic, the total magnetic flux density in the ferromagnetic bodies is $\underline{\boldsymbol{B}} = \underline{\boldsymbol{B}}^J + \underline{\boldsymbol{B}}^{J_i} + \underline{\boldsymbol{B}}^{I}$ where $\underline{\boldsymbol{B}}^J$, $\underline{\boldsymbol{B}}^{J_i}$, and $\underline{\boldsymbol{B}}^{I}$ are produced by the induced currents, the imposed currents and the magnetic polarization [8], respectively

$$\underline{\boldsymbol{B}}^J(\boldsymbol{r}) = \gamma\;\boldsymbol{k}\times\int_{\Omega}\frac{\underline{J}(\boldsymbol{r}')\boldsymbol{R}}{R^2}ds' \tag{6}$$

$$\underline{\boldsymbol{B}}^{J_i}(\boldsymbol{r}) = \gamma\;\boldsymbol{k}\times\int_{\Omega_i}\frac{\underline{J}_i(\boldsymbol{r}')\boldsymbol{R}}{R^2}ds' \tag{7}$$

$$\underline{\boldsymbol{B}}^{I}(\boldsymbol{r}) = \frac{1}{2\pi}\int_{\Omega_f}\frac{(\nabla'\times\underline{\boldsymbol{I}}(\boldsymbol{r}'))\times\boldsymbol{R}}{R^2}ds'. \tag{8}$$

The total magnetic flux density $\boldsymbol{B}(t)$ in the time domain is evaluated, similar to (4), as the sum of its N harmonics. At selected times correction (1) [6] is performed. Then the Fourier series of $\boldsymbol{I}$ is computed again. To reduce the electromagnetic field computation time, at first, only the fundamental in (4) is retained. Then, for an increased accuracy, the 3rd and 5th harmonics are added gradually, few iterations being required for PFPM. The fewer the harmonics are used, the faster the convergence speed is. Moreover, when only the fundamental is retained, the correction $\underline{\boldsymbol{B}} \rightarrow \underline{\boldsymbol{H}}$ can be done directly, and a dynamic overrelaxation procedure can be used. This procedure radically speeds up the convergence [7].

The magnetic force average value over a period is the sum of its harmonics components average values

$$\boldsymbol{F}_{\mathrm{av}}^{\mathrm{tot}} = \boldsymbol{k}\times\sum_{n=1,3,\ldots}\mathrm{Re}\left(\int_{\Omega}\underline{J}_n^*(\boldsymbol{r}')\underline{\boldsymbol{B}}_n(\boldsymbol{r}')\,ds'\right) \tag{9}$$

where the asterisk indicates the complex conjugate and the magnetic flux density has two components: one produced by the polarizations of ferromagnetic bodies and the other by the imposed currents.

V. Numerical Computations

A. Discretization of the Current Density Integral Equation

The domain Ω is divided into M subdomains ω_m, having constant current densities, the domain Ω_i is divided into M_i subdomains ω_{i_k}, having constant imposed current densities, and the domain Ω_f is divided into M_f subdomains ω_{f_k}, having constant polarization. Integrating (5) for every harmonic over each ω_m gives

$$\begin{aligned}
A\underline{J} + j\lambda B\underline{J} &= -j\lambda B_i\underline{J}_i + \gamma\,(\boldsymbol{v}\cdot\boldsymbol{C}_i)\underline{J}_i + jnf(\boldsymbol{k}\times\boldsymbol{C}_f)\cdot\underline{\boldsymbol{I}}_f \\
&\quad + \left((\boldsymbol{k}\times\boldsymbol{v})\overline{\overline{D}}_f\right)\cdot\underline{\boldsymbol{I}}_f.
\end{aligned} \tag{10}$$

The diagonal matrix A has entries $A_m = \rho_m S_m$. The vectors $\underline{J}$, $\underline{J}_i$, and $\underline{\boldsymbol{I}}_f$ have entries $\underline{J}_m$, $\underline{J}_{i_m}$, and $\underline{\boldsymbol{I}}_{f_m}$, respectively. In each subdomain, the entries of matrices B, B_i, $\boldsymbol{C}_i$, and $\boldsymbol{C}_f$ are

$$\begin{aligned}
B_{m,k} &= \int_{\omega_m}\int_{\omega_k}\ln(1/R)\,ds_k ds_m \\
B_{i_{m,k}} &= \int_{\omega_m}\int_{\omega_{i_k}}\ln(1/R)\,ds_k ds_m \\
\boldsymbol{C}_{i_{m,k}} &= \int_{\omega_m}\int_{\omega_{i_k}}(\boldsymbol{R}/R^2)ds_k ds_m \\
\boldsymbol{C}_{f_{m,k}} &= \int_{\omega_m}\int_{\omega_{f_k}}(\boldsymbol{R}/R^2)ds_k ds_m.
\end{aligned}$$

The entries of $\overline{\overline{D}}_f$ are

$$\overline{\overline{D}}_{f_{m,k}} = -(1/2\pi)\int_{\partial\omega_m}\int_{\partial\omega_{f_k}}\ln R(d\boldsymbol{l}_m d\boldsymbol{l}_k)$$

where $(d\boldsymbol{l}_m d\boldsymbol{l}_k)$ is the dyad formed by the vector length elements $d\boldsymbol{l}_m$ and $d\boldsymbol{l}_k$. If the distance between the weight centers of subdomains ω_m, ω_k is large enough, simplified approximate relationships could be used for a shorter computation time of matrices B, B_i, $\boldsymbol{C}_i$, $\boldsymbol{C}_f$, and $\overline{\overline{D}}_f$.

B. Magnetic Flux Density Computation

From (6) to (8), the space average magnetic flux densities in subdomain ω_{f_m}, due to induced and imposed current densities and to magnetic polarizations are

$$\underline{\tilde{\boldsymbol{B}}}_{f_m}^{J} = \gamma\frac{1}{S_{f_m}}\boldsymbol{k}\times\sum_{k=1}^{M}\boldsymbol{C}_{f_{k,m}}\underline{J}_k \tag{11}$$

$$\underline{\tilde{\boldsymbol{B}}}_{f_m}^{J_i} = \gamma\frac{1}{S_{f_m}}\boldsymbol{k}\times\sum_{k=1}^{M_i}\boldsymbol{C}_{(f,i)_{k,m}}\underline{J}_{i_k} \tag{12}$$

$$\underline{\tilde{\boldsymbol{B}}}_{f_m}^{I} = \frac{1}{S_{f_m}}\sum_{k=1}^{M_f}\overline{\overline{D}}_{(f,f)_{m,k}}\underline{\boldsymbol{I}}_{f_k}. \tag{13}$$

with

$$\boldsymbol{C}_{(f,i)_{k,m}} = \int_{\omega_{fm}} \int_{\omega_{i_k}} (\boldsymbol{R}/R^2) ds_k ds_m$$

$$\overline{\overline{D}}_{(f,f)_{m,k}} = -(1/2\pi) \int_{\partial\omega_{fm}} \int_{\partial\omega_{f_k}} \ln R (d\boldsymbol{l}_m d\boldsymbol{l}_k).$$

The magnetic flux density in the ferromagnetic bodies is computed, in matrix form, as

$$\underline{\tilde{\boldsymbol{B}}}_f = \underline{\tilde{\boldsymbol{B}}}_f^{J_i} + S_f^{-1} \overline{\overline{D}}_{f,f} \underline{\boldsymbol{I}}_f + \gamma \; S_f^{-1} (\boldsymbol{k} \times \boldsymbol{C}_f^T) \underline{J}. \tag{14}$$

C. Computation of the Average Magnetic Force Given by Each Harmonic

The average magnetic force over a period, for each harmonic n, is evaluated using

$$\boldsymbol{F}_{\text{av}} = \gamma \; \boldsymbol{k} \times \sum_{m=1}^{M} \text{Re} \left(\underline{J}_m^* \sum_{k=1}^{M_i} (\boldsymbol{k} \times \boldsymbol{C}_{i_{m,k}}) \underline{J}_{i_k} \right) + \boldsymbol{k} \times \sum_{m=1}^{M} \text{Re} \left(\underline{J}_m^* \sum_{k=1}^{M_f} \overline{\overline{D}}_{f_{m,k}} \underline{\boldsymbol{I}}_{f_k} \right). \tag{15}$$

Entries $\boldsymbol{C}_{i_{m,k}}$ and $\overline{\overline{D}}_{f_{m,k}}$ are computed only once, at the first assembly of the eddy current integral equation. If the imposed current density has only the fundamental component, the first term in (15) is zero for $n > 1$.

VI. Equation of Motion

The motion equation is

$$m \frac{d^2 \boldsymbol{r}}{dt^2} = \boldsymbol{F}\left(\boldsymbol{r}, \frac{d\boldsymbol{r}}{dt}, J^i, \boldsymbol{I}\right) + \boldsymbol{G} \tag{16}$$

where m is the mass of the conducting body and $\boldsymbol{G}$ is its gravitational force. The force $\boldsymbol{F}$ is due to the magnetic field and is a function of the conducting body position $\boldsymbol{r}$, its velocity $\boldsymbol{v} = d\boldsymbol{r}/dt$, the excitation current density J^i, and the magnetic polarization $\boldsymbol{I}$. Equation (16) is solved iteratively, following the scheme described in [5]. Knowing the given position vector $\boldsymbol{r}_1$ and velocity $\boldsymbol{v}_1$, the magnetic force $\boldsymbol{F}_1$ at time t_1 is computed with (9). The numerical solution of (16) for the interval $[t_1, t_2 = t_1 + \Delta t]$ is obtained, at first, by assuming the magnetic force at time $t_2 = t_1 + \Delta t$ is $\boldsymbol{F}_2^{(0)} = \boldsymbol{F}_1$, with Δt being sufficiently small.

Assuming a linear variation of the magnetic force during the given time interval $[t_1, \; t_2]$

$$\boldsymbol{F}(t) = \boldsymbol{F}_1 + \left(\boldsymbol{F}_2^{(0)} - \boldsymbol{F}_1\right) \frac{t - t_1}{\Delta t} \tag{17}$$

the velocity $\boldsymbol{v}_2$ and the position vector $\boldsymbol{r}_2$ at the time t_2 are

$$\boldsymbol{v}_2 = \boldsymbol{v}_1 + \left(\boldsymbol{F}_2^{(0)} + \boldsymbol{F}_1\right) \frac{\Delta t}{2m} \tag{18}$$

$$\boldsymbol{r}_2 = \boldsymbol{r}_1 + \boldsymbol{v}_1 \Delta t + \left(\boldsymbol{F}_2^{(0)} + 2\boldsymbol{F}_1\right) \frac{\Delta t^2}{6m}. \tag{19}$$

Now, the electromagnetic field problem can be solved and the value of the magnetic force $\boldsymbol{F}_2^{(1)}$ at the time t_2 can be obtained. The new value $\boldsymbol{F}_2^{(1)}$ replaces $\boldsymbol{F}_2^{(0)}$ in (17), and the operations in (18) and (19) are repeated. The magnetic force at time the t_2 is computed repeatedly, until the difference between its successive values is small enough. Then, the computations proceeds to the next time step.

A time step of $\Delta t = T$ is chosen for the motion problem.

VII. Computation Scheme

The following algorithm is proposed:

1) The following matrices and vectors are computed for the fundamental and a few harmonics: $\underline{\tilde{\boldsymbol{B}}}_f^{J_i}$, $\overline{\overline{D}}_{(f,f)}$, B, $Q = A^{-1} B A^{-1}$, and $T = (1 + \lambda^2 A^{-1} B A^{-1} B)^{-1}$.
2) Knowing the given position vector $\boldsymbol{r}$ and velocity $\boldsymbol{v}$, the following matrices are computed: B_i, $\boldsymbol{C}_i$, $\boldsymbol{C}_f$, $\overline{\overline{D}}_f$, $W' = -\lambda B_i J_i^{\text{re}} + \gamma (\boldsymbol{v} \cdot \boldsymbol{C}_i) J_i^{\text{im}}$, and $W'' = \lambda B_i J_i^{\text{im}} + \gamma (\boldsymbol{v} \cdot \boldsymbol{C}_i) J_i^{\text{re}}$.
3) The induced currents are computed by solving the eddy current integral equation and applying the PFPM. At first, the final value of the magnetic polarization from the previous position is used for computing

$$L' = W' + \frac{\omega}{2\pi} (\boldsymbol{k} \times \boldsymbol{C}_f) \cdot \boldsymbol{I}_f^{\text{re}} + ((\boldsymbol{k} \times \boldsymbol{v}) \overline{\overline{D}}_f) \cdot \boldsymbol{I}_f^{\text{im}}$$

$$L'' = W'' - \frac{\omega}{2\pi} (\boldsymbol{k} \times \boldsymbol{C}_f) \cdot \boldsymbol{I}_f^{\text{im}} + ((\boldsymbol{k} \times \boldsymbol{v}) \overline{\overline{D}}_f) \cdot \boldsymbol{I}_f^{\text{re}}.$$

The induced current is computed using (10) as

$$J^{\text{re}} = T(\lambda Q L' + A^{-1} L''), \quad J^{\text{im}} = A^{-1}(-\lambda B J^{\text{re}} + L').$$

The average magnetic flux densities in the ferromagnetic bodies are computed using (14).

When considering only the fundamental, the magnetic polarization in the ferromagnetic bodies can be corrected, in the frequency domain, by direct computation $\underline{\tilde{\boldsymbol{B}}}_f \rightarrow \underline{\boldsymbol{I}}_f$. When additional harmonics are considered, the values of $\tilde{\boldsymbol{B}}_f(t)$ are computed over a period, the magnetic polarization is corrected using (1) and the Fourier expansion of $\boldsymbol{I}_f$ is, then, computed. If the difference between the corrected magnetic polarization and its initial value is not small enough, one returns to step 3.
4) The initial value of the force $\boldsymbol{F}_1$ is evaluated using (9).
5) The new position and velocity are obtained using (18) and (19) choosing $\boldsymbol{F}_2^{(0)} = \boldsymbol{F}_1$. Force $\boldsymbol{F}_2^{(1)}$ is computed using (15) by going back to steps 2 and 3. Using (17)–(19) position $\boldsymbol{r}$ and velocity $\boldsymbol{v}$ at time $t_2 = t_1 + \Delta t$ are corrected. If the difference between the new and the old position is not small enough, one returns to step 2.

VIII. Simulation Example

The ferromagnetic circuit (Fig. 1) has the magnetic characteristic as shown in Fig. 2. Its columns have the height of 40 mm and the widths of 10, 20, and 10 mm, respectively. The levitated copper plate with resistivity of 2×10^{-8} Ωm is located above them and has a 80 mm × 4 mm cross section. The lower yoke has the height of 10 mm and width of 80 mm. The magnetic circuit is made of very thin shields or of ferrite, and therefore its eddy currents can be neglected.

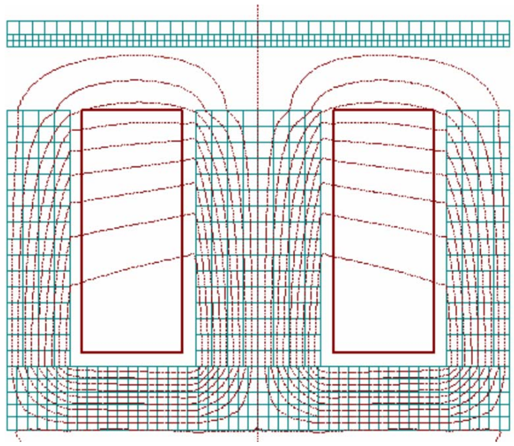

Fig. 1. Levitation device structure and the discretization mesh.

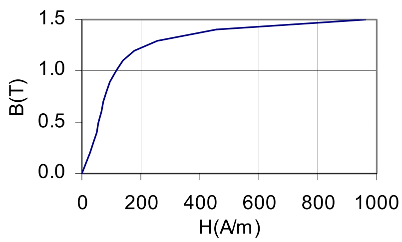

Fig. 2. B–H magnetic characteristic.

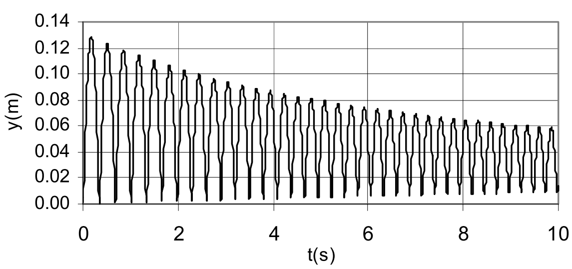

Fig. 3. Evolution in time of the coordinate y of the plate for $f = 200$ Hz.

The coil around the central column has a 16 mm × 38 mm rectangular cross section. The imposed current of the coil is sinusoidal with a RMS value $I = 3000$ A turn and a frequency $f = 200$ Hz. The copper plate is located 10 mm above the magnetic circuit, in the origin of the Oz axes, having only a vertical degree of freedom. The initial velocity is zero. The vertical displacement of the levitating plate is shown in Fig. 3.

IX. Conclusion

Without the magnetic circuit, the average value of the levitating force is 20.8378 N/m, in the upward direction. With the magnetic circuit, the electromagnetic field is initially computed only for the fundamental, adding successively the 3rd and 5th harmonics [6]. The average forces values due to the fundamental and the two harmonics are 414 N/m, 4×10^{-6} N/m and 8.5×10^{-6} N/m, respectively. The large differences between the contribution of the fundamental and those of the harmonics allow computing only the contribution of the fundamental. In this case, one can use the method of dynamic overrelaxation presented in [7]. Total computation time was 2826 s, on a notebook with 2.5 GHz CPU speed.

Appendix

The magnetic flux density $\boldsymbol{B}^{J_i}$ due to the imposed current density verifies

$$\boldsymbol{k} \cdot (\boldsymbol{v} \times \boldsymbol{B}^{J_i}) = \boldsymbol{k} \cdot \left(\boldsymbol{v} \times \gamma \int_{\Omega_i} J_i \boldsymbol{k} \times \frac{\boldsymbol{R}}{R^2} ds' \right) = \gamma \int_{\Omega_i} J_i \boldsymbol{v} \cdot \frac{\boldsymbol{R}}{R^2} ds' \tag{A1}$$

because in 2-D structures

$$\boldsymbol{k} \cdot [\boldsymbol{v} \times (\boldsymbol{k} \times \boldsymbol{R})] = \boldsymbol{v} \cdot \boldsymbol{R} - (\boldsymbol{k} \cdot \boldsymbol{R}) \underbrace{(\boldsymbol{k} \cdot \boldsymbol{v})}_{0}.$$

The magnetic flux density $\boldsymbol{B}^{I}$ due to the magnetic polarization verifies

$$\begin{aligned} \boldsymbol{k} \cdot (\boldsymbol{v} \times \boldsymbol{B}^{I}) &= \boldsymbol{k} \cdot \left(\boldsymbol{v} \times \frac{1}{2\pi} \int_{\Omega_f} \frac{(\nabla' \times \underline{\boldsymbol{I}}(\boldsymbol{r}')) \times \boldsymbol{R}}{R^2} ds' \right) \\ &= \frac{1}{2\pi} \boldsymbol{k} \cdot \int_{\Omega_f} \left(\boldsymbol{v} \cdot \frac{\boldsymbol{R}}{R^2} \right) \left(\nabla' \times \boldsymbol{I}(\boldsymbol{r}', t) \right) ds'. \end{aligned} \tag{A2}$$

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