Evaluation of Magnetic Material Losses Produced by Hysteresis and Eddy Currents

Valentin Ionita and Gabriel Preda

Facultatea Electrotehnica, Universitatea "Politehnica" Bucuresti
Spl. Independentei-313, 77206 Bucharest, ROMANIA

Abstract— **The paper presents a fast method for computing the quasi-stationary electromagnetic field in devices containing both magnetically nonlinear materials and hysteretic materials. The linear iterative procedure of fixed-point type allows the correct evaluation of the local field and of the losses produced by hysteresis and eddy currents. The tests are performed on a single-sheet device used to determine the magnetic material losses under the a.c. uniform field.**

Index Terms— **Losses, hysteresis, eddy currents**

INTRODUCTION

The electromagnetic device optimization with CAD methods requires a compromise between the accuracy and the simplicity of the physical models. The modeling of eddy currents and hysteresis for magnetic materials in variable fields influences both the computing time (by the model complexity) and the efficiency of the classical numerical methods (e.g. the Newton-Raphson method, usually chosen for nonlinear media, may be divergent for hysteretic media [1]).

The paper presents a fast method for computing the quasi-stationary electromagnetic field in devices containing both magnetically nonlinear materials and hysteretic materials. The linear iterative procedure of fixed-point type, presented in [2], allows a sure determination of the correct numerical solution. The control of the iterative solution accuracy allows the correct evaluation of the local field and of the losses produced by hysteresis and eddy currents. Any dynamic vector hysteresis model that is locally Lipschitzian and monotonously uniform [3] may be used. The tests are performed on a single-sheet device used to determine the magnetic material losses under the a.c. uniform field.

PROBLEM FORMULATION

Let us consider the plan-parallel domain $\Omega\subseteq\Re^2$ (Fig. 1), composed by the hysteretic domain Ω_H, the magnetically nonlinear domain Ω_N and the non-conductive domain Ω_I in which the known density $\vec{J}$ of the supplied currents is normal on Ω.

Manuscript received November 3, 1997.

V. Ionita, fax (and phone) 40-1-3121190, vali@lmn.pub.ro; G. Preda, preda@lmn.pub.ro

This work was supported in part by the MCT-836 grant

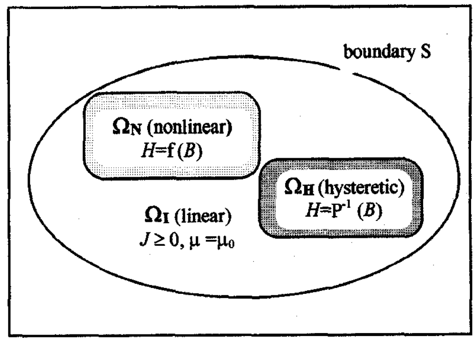

Fig. 1. Computation domain

The eddy current problem is described by the quasi-stationary regime equations:

- in Ω_H and Ω_N:

$$\begin{cases}\text{curl}\,\vec{H}=\vec{J}_M\;;\;\text{div}\,\vec{B}=0\;;\;\text{curl}\,\vec{E}=-\dfrac{\partial\vec{B}}{\partial t}\;;\;\vec{J}_M=\sigma\vec{E}\\ \vec{H}=\vec{H}(\vec{B}),\ \text{including the hysteresis phenomenon for } \Omega_H\end{cases}\qquad(1)$$

- in Ω_I: $\text{curl}\,\vec{H}=\vec{J}\,;\ \text{div}\,\vec{B}=0\,;\ \vec{H}=\nu_0\vec{B}$ (2)

Zero initial conditions and the boundary conditions (H_t or B_n are known on the boundary S) are also considered.

COMPUTATION METHOD

The modified magnetic vector potential $\vec{A}^*$ is chosen as the problem unknown. The time derivative is eliminated by a Crank-Nicolson algorithm, so a stationary problem has to be solved at each instant t_{k+1}. Each problem of this type is treated according to the fixed-point iterative method presented in [2]. The nonlinear material and the hysteretic one are replaced by a virtual medium described by the linear model:

$$\vec{H}=\nu\vec{B}-\vec{I}\qquad(3)$$

where $\vec{I}$ is an iterative corrected source, obtained by applying the real material relationship (nonlinear in Ω_N and hysteretic in Ω_H) to the computed flux density $\vec{B}$. The conventional reluctivity ν is chosen in order to assure the convergence of iterative procedure [4]. The convergence rate may be improved by choosing different values of ν for the

0018-9464/98$10.00 © 1998 IEEE

nonlinear domain and the hysteretic one, according to the real magnetic characteristics.

The associated linear problem for each iteration and for each time step t_{k+1} is described by the following equations:

$$\begin{cases} \frac{\Delta t}{2}\left[\mathrm{curl}\left(\nu\,\mathrm{curl}\vec{A}^*_{k+1}\right)\right] + \sigma\vec{A}^*_{k+1} = \frac{\Delta t}{2}\left[-\mathrm{curl}\left(\nu\,\mathrm{curl}\vec{A}^*_{k}\right)\right] + \sigma\vec{A}^*_{k} + \\ \quad \frac{\Delta t}{2}\left[\mathrm{curl}\ \nu\vec{I}_{k+1} + \mathrm{curl}\ \nu\vec{I}_{k}\right] \quad \text{in } \Omega_H \text{ and } \Omega_N \\ \left[\mathrm{curl}\left(\nu_0\ \mathrm{curl}\vec{A}^*_{k+1}\right)\right] = \vec{J}_{k+1} + \vec{J}_{k} - \mathrm{curl}\left(\nu_0\ \mathrm{curl}\vec{A}^*_{k}\right) \quad \text{in } \Omega_I \end{cases} \tag{4}$$

Δt being the constant time step. This linear problem is solved by finite elements method (Galerkin), using linear shape functions.

The choice of a constant reluctivity ν all over the domain Ω_M allows the obtaining of the same matrix for each iteration and for each time step. So, this matrix can only be computed and preconditioned once, at the beginning of the iterations; this advantage of the proposed method, combined with an optimal choice of ν allows to reduce the involved CPU time.

The constitutive hysteretic dependence $\vec{H}(\vec{B})$ is implemented by a model which considers a strong uniaxial anisotropy. So, the $\vec{B}$ and $\vec{H}$ projections on the easy magnetization axis (e) and on the normal direction (h) satisfy:

$$H_e = \mathrm{P}^{-1}(B_e) \tag{5}$$

$$H_h = \nu_0\, B_h \tag{6}$$

where P^{-1} denotes the inverse of the scalar Preisach hysteresis model (identified by Biorci-Pescetti procedure [5]) and ν_0 is the vacuum reluctivity. The easy magnetization axis (e) may be different for every component of the device. In single-sheet device analysis this simple model is useful and efficient enough.

Losses Evaluation

After determining the magnetic field magnitudes, the magnetic material losses are computed by numerical integration, separately for the two components involved by static hysteresis (P_H) and eddy currents (P_F).

$$P_H = \frac{1}{\gamma T}\int_0^T \vec{H}\,\mathrm{d}\vec{B} \tag{7}$$

$$P_F = \frac{1}{\gamma T}\int_0^T \vec{E}\cdot\vec{J}\,\mathrm{d}t = \frac{1}{\gamma T}\int_0^T \sigma\left(\frac{\partial A^*}{\partial t}\right)^2 \mathrm{d}t \tag{8}$$

where γ is the material density and T is the analyzed time interval. Considering N intervals $[t_k, t_{k+1}]$ of T and a linear evolution in time of the field magnitudes for each interval, the previous relations become:

$$P_H = \frac{1}{2\gamma T}\sum_{k=1}^{N}\left(H_k + H_{k+1}\right)\left(B_{k+1} - B_k\right) \tag{9}$$

$$P_F = \frac{1}{\gamma T}\sum_{k=1}^{N}\sigma\left(\frac{A^*_{k+1} - A^*_k}{\Delta t}\right)^2 \cdot \Delta t \tag{10}$$

For the particular case of uniform field created by an a.c. applied current into a sheet, these losses may be computed with the formula:

$$P_H^{(u)} = \frac{W_H}{\gamma T} \tag{11}$$

$$P_F^{(u)} = \frac{\pi^2}{6\gamma T^2}\cdot\sigma\cdot B_{\max}^2\cdot d^2 \tag{12}$$

where W_H is the hysteresis cycle surface (according the Warburg theorem), d is the sheet thickness and $B_{\max}$ is the sinusoidal flux density amplitude. It is clear that $P_H = P_H^{(u)}$ because the used hysteresis model was a static one.

Numerical Application

The single-sheet device of Fig. 2 is used for losses measurement under a uniform field, created by Helmholtz type coils supplied with a.c. current i_e. The easy magnetization axis (e) of the hysteretic material corresponds to the Ox axis. The sheet dimensions are (in mm) 60x60x0.35 and the measurement coils positions are fixed in order to obtain a minimum error [6]. The experimental device assures a total error ε=5.6%.

The tests were performed both for hard ferromagnetic materials (Alnico with B_r=1.3T, H_c=500A/m) and for soft ferrite (B_r=0.18T, H_c=40A/m), while the yoke (Fe-Si sheets) is always nonlinear. The results fit well to the experimental data. For example, the magnetic field distribution for an Alnico sheet, corresponding to the i_e=0 moment, are presented in Fig. 3. Table I also indicates the measured and computed average losses for 1Hz, 50Hz and 500Hz, the source current being controlled in order to assure the achievement of the major hysteresis cycle. The results can be compared in Fig. 4 for the ferrite.

TABLE I

Measured Losses (P_m), Hysteresis Computed Losses (P_H) and Eddy Current Computed Losses (P_F, $P_F^{(u)}$) for Soft Ferrite (B_{max}=0.39 T) and Alnico (B_{max}=1.46 T)

Material	P_m [W/kg]	P_H [W/kg]	P_F [W/kg]	$P_F^{(u)}$ [W/kg]
Soft Ferrite				
- at 1 Hz	7.1 E-3	6.02 E-3	1.53 E-3	1.36 E-3
- at 50 Hz	0.3	0.324	0.0392	0.0365
- at 500 Hz	6.8	3.288	2.85	2.71
Alnico				
- at 1 Hz	-	0.341	2.28 E-4	2.18 E-4
- at 50 Hz	-	17.194	0.54	0.39
- at 500 Hz	-	172.7	39.1	38.05

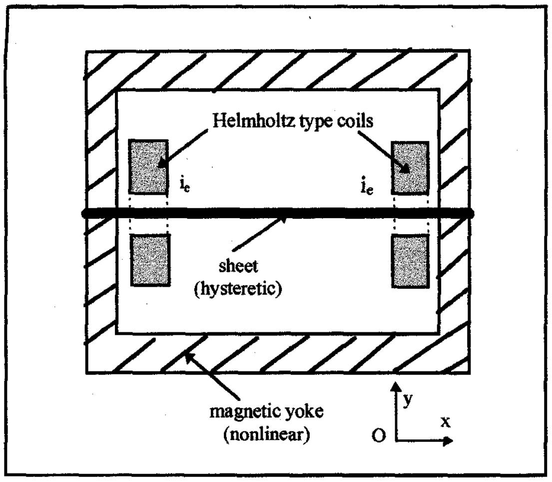

Fig. 2. Single-sheet device

The peek values B_{max} of the magnetic flux density was limited by the device saturation which alters the measurement coils data accuracy. At the same time, the device design allows to obtain a uniform magnetic field into the sheet, so the values computed with (9), (10) and (11), (12) must be similar. However, some numerical differences can appear in the corner regions, as the result of the vector dynamic hysteresis modeling with a scalar static model.

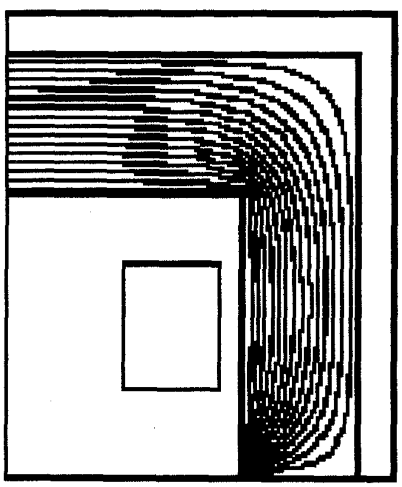

Fig. 3. Magnetic field distibution for Alnico sheet at f=50 Hz and i_e=0

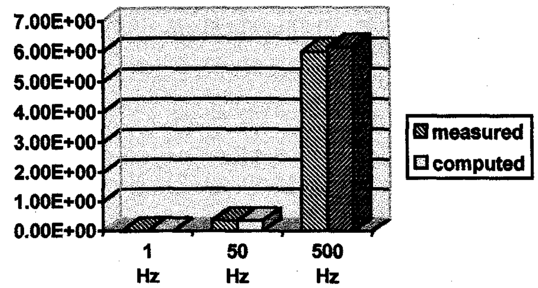

Fig.4. Measured and computed losses for a ferrite

Conclusions

The method allows a fast and sure evaluation of losses in magnetic sheets under steady-state uniform field. The fixed-point type iterative procedure treats both the sheet hysteresis and the magnetic yoke nonlinearity. The optimization of Helmholtz-type coils position by numerical simulation involves a uniform distribution of losses on the entire sheet. The considered hysteresis model is adapted to the particular geometry of the single-sheet device and furnishes correct results in a short time, for any frequency for which the dynamic hysteresis can be neglected.

The proposed method may be also used for any electromagnetic device containing nonlinear and hysteretic magnetic materials with a geometry which allows the scalar hysteresis model using.

References

[1] B.B. Shyamkumar and Z.J. Cendes, "Convergence of iterative methods for nonlinear magnetic field problems", *IEEE Transactions on Magnetics*, vol. 24, pp. 2585-2587, Nov. 1988

[2] V. Ionita, B. Cranganu and D. Ioan, "Quasi-stationary magnetic field computation in hysteretic media", *IEEE Transactions on Magnetics*, vol. 32, pp. 1128-1131, May 1996

[3] J.M. Ortega and W.C. Rheinboldt, *Iterative Solution of Nonlinear Equations in Several Variables*, Academic Press, New York, pp. 384-390, 1970

[4] V. Ionita and F. Ossart, "Etude de la convergence de la methode du point fixe pour calculer le champ magnetique dans les materiaux hysteretiques", *NUMELEC'97* Proceedings, pp.. 50-51, Lyon-France, April 1997

[5] G. Biorci and D. Pescetti, "Analytical theory of the behavior of ferromagnetic materials", *Il Nuovo Cimento*, no. 6, pp. 830-842, 1958

[6] T. Nakata, N. Takahashi, K. Fujiwara, M. Nakano and T. Kayada, "Effects of eddy currents in the specimen in a single sheet tester on measurement errors", *IEEE Transactions on Magnetics*, vol. 26, pp. 1641-1645, Sept. 1990