A Nonlinear Eddy-Current Integral Formulation for Moving Bodies

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***Abstract*- This paper presents an integral formulation for the calculation of the eddy-current problems in moving conductors in the presence of magnetic media. The quasistationary Maxwell equations are written in local reference frames associated with moving bodies. Only the conducting and ferromagnetic domains are discretized. The eddy current is described in terms of a two component electric vector potential for which edge elements are used along with the tree-cotree decomposition. The magnetization is assumed to be uniform in each element of the ferromagnetic domain. Time stepping is used for time integration. The nonlinear problem is solved using Picard iteration, for which convergence is guaranteed. Only a part of the relevant matrices must be formed at each time step. The features of the method are illustrated with the aid of some numerical results.**

Index terms – **Eddy currents, integral equations, finite element methods, nonlinear magnetics, moving conductors.**

I. Introduction

The eddy current analysis in moving conductors is of interest both in the case of rigid motion (especially for electrical machines) and deformable bodies (e.g., fluid conductors, study of vibrations, and so on).

Two main classes of methods have been proposed for the calculation of the electromagnetic field in moving bodies: differential and integral formulations.

The differential methods show a number of disadvantages. The number of unknowns needed to obtain a desired accuracy is rather huge, the analysis of unbounded domains requires a special treatment, and the mesh nodal coordinates and incidence matrices must be modified during the time evolution. Differential formulations have been proposed in terms of the magnetic vector potential [1,2] or scalar potentials [3]. In the case of deformable conductors, as suggested in [4], it is possible to take advantage of lagrangian approach and differential geometry.

As stated in [5], the integral methods appear more promising. They are characterized by full matrices, but the number of unknowns needed to get a required accuracy is relatively small. In addition, the regularity conditions at infinity are automatically taken into account by the formulation. Finally, in the case of rigid motion, the topology of the mesh is not modified, and only a part of the stiffness matrices must be changed during the movement. The integral formulation proposed in [6] is an extension of the approach presented in [7] to take into account the presence of linear moving media. A similar approach is presented in [8].

The possibility of including magnetic materials into the models with fixed conductors has already been discussed by several authors using different approaches [9-15].

In this paper, the methods proposed in [7] and [12] are combined to take into account the presence of non-linear moving media.

II. Problem Formulation

The model is based on the magnetoquasistatic limit of Maxwell equations and the following constitutive relationships:

$$\mathbf{E} = \eta \mathbf{J} \qquad \text{in } V_c \tag{1}$$

$$\mathbf{H} = f(\mathbf{B}) \qquad \text{in } V_f \tag{2}$$

where $\mathbf{J}$ is the current density, $\mathbf{E}$ is the electric field, η is the resistivity, $\mathbf{H}$ is the magnetic field $\mathbf{M}$ is the magnetization vector, $\mathbf{B}$ is the flux density, V_c is the conducting domain and V_f is the ferromagnetic domain.

Constitutive equation (2) is equivalent to:

$$\mathbf{B} = \mu_0 (\mathbf{H}+\mathbf{M}) \tag{3}$$

$$\mathbf{M} = g(\mathbf{B}) = \nu_0 \mathbf{B} - f(\mathbf{B}) \tag{4}$$

where μ_0 is the vacuum permeability and $\nu_0=1/\mu_0$. We assume f to be uniformly monotonic and verify Lipschitz condition.

For the sake of simplicity we assume zero initial conditions for all fields.

We consider a set of N conducting and/or ferromagnetic bodies B_k, each of them moving with a rigid velocity $\mathbf{v}_k$. In

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the local frame of any moving body the time integral of the electric field is given by:

$$\int_0^t \mathbf{E}(\mathbf{x},\tau)d\tau = -\mathbf{A}(\mathbf{x},t) - \nabla V \quad (5)$$

where V is the time integral of the scalar electric potential and $\mathbf{A}$ is the divergence-free magnetic vector potential given by:

$$\mathbf{A}(\mathbf{x},t) = \frac{\mu_0}{4\pi}\int_{V_c}\frac{\mathbf{J}(\mathbf{x}',t)}{|\Delta\mathbf{x}(t)|}dV' + \frac{\mu_0}{4\pi}\int_{V_f}\frac{\mathbf{M}(\mathbf{x}',t)\times\Delta\mathbf{x}(t)}{|\Delta\mathbf{x}(t)|^3}dV' + \mathbf{A}_0(\mathbf{x},t) \quad (6)$$

where $\Delta\mathbf{x}=\mathbf{x}-\mathbf{x}'$ and $\mathbf{A}_0$ is the contribution of the external current density $\mathbf{J}_s$ in the volume V_0 moving with velocity $\mathbf{v}_s$:

$$\mathbf{A}_0(\mathbf{x},t) = \frac{\mu_0}{4\pi}\int_{V_0}\frac{\mathbf{J}_s(\mathbf{x}',t)}{|\Delta\mathbf{x}(t)|}dV' \quad (7)$$

The magnetic field $\mathbf{B}=\nabla\times\mathbf{A}$ can directly be obtained using Biot-Savart law.

With this choice the Maxwell equations in the moving frame are automatically satisfied. Therefore only the constitutive equations (1) and (4) must explicitly be imposed.

III. Numerical Approach

The current density $\mathbf{J}$ is expressed in terms of cotree edge shape functions $\mathbf{T}_k$ [7]:

$$\mathbf{J}(\mathbf{x},t) = \sum_{k=1}^{n} \mathrm{I}_k(t)\nabla\times\mathbf{T}_k \quad (8)$$

whereas the magnetization $\mathbf{M}$ is approximated as a piecewise uniform field:

$$\mathbf{M}(\mathbf{x},t) = \sum_{k=1}^{n_m} \mathrm{M}_k(t)\mathbf{P}_k \quad (9)$$

where $\mathbf{P}_k$'s are n_m unit vector pulse functions obtained by multiplying the unit vectors along the coordinate axes by the usual scalar unit pulse functions p_k's, which are different from zero only for $\mathbf{x}$ belonging to the kth finite element of V_f.

We apply Galerkin approach in the following form:

$$\int_0^t\int_{V_c}\nabla\times\mathbf{T}_i\cdot(\mathbf{E}-\eta\mathbf{J})dV = 0 \quad \forall\mathbf{T}_i \quad (10)$$

$$\int_{V_f}\mathbf{P}_i\cdot[g^{-1}(\mathbf{M})-\mathbf{B}]dV = 0 \quad \forall\mathbf{P}_i \quad (11)$$

obtaining the system of nonlinear equations:

$$\{L\}[I] + \{F\}[M] + \{R\}\int_0^t[I(\tau)]d\tau = [U] \quad (12)$$

$$\{D\}\,G^{-1}([M]) - \{E\}[M] - \{F\}^T[I] = [W] \quad (13)$$

where:

$$L_{ij} = \frac{\mu_0}{4\pi}\int_{V_c}\int_{V_c}\frac{\nabla\times\mathbf{T}_i\cdot\nabla\times\mathbf{T}_j}{|\mathbf{x}-\mathbf{x}'|}dVdV' \quad (14)$$

$$R_{ij} = \int_{V_c}\nabla\times\mathbf{T}_i\cdot\eta\nabla\times\mathbf{T}_j dV \quad (15)$$

$$U_i = -\frac{\mu_0}{4\pi}\int_{V_c}\int_{V_0}\frac{\nabla\times\mathbf{T}_i(\mathbf{x})\cdot\mathbf{J}_S(\mathbf{x}',t)}{|\mathbf{x}-\mathbf{x}'|}dVdV' \quad (16)$$

$$F_{ij} = \frac{\mu_0}{4\pi}\int_{V_c}\int_{V_f}\frac{\nabla\times\mathbf{T}_i(\mathbf{x})\cdot\mathbf{P}_j(\mathbf{x}')\times(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}dVdV' \quad (17)$$

$$D_{ij} = \int_{V_f}\mathbf{P}_i\cdot\mathbf{P}_j\,dV \quad (18)$$

$$E_{ij} = \mu_0\int_{V_f}\mathbf{P}_i\cdot\mathbf{P}_j dV - \frac{\mu_0}{4\pi}\int_{\partial V_{fi}}\int_{\partial V_{fj}}\frac{\mathbf{n}\cdot\mathbf{P}_i(\mathbf{x})\,\mathbf{n}'\cdot\mathbf{P}_j(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}dSdS' \quad (19)$$

$$W_i = \frac{\mu_0}{4\pi}\int_{\Re^3-V_c}\int_{V_f}\frac{\mathbf{P}_i(\mathbf{x})\cdot\mathbf{J}_S(\mathbf{x}',t)\times(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}dVdV' \quad (20)$$

$$G^{-1}([M]) = \{D\}^{-1}\int_{V_f}\mathbf{P}_i\cdot[g^{-1}(\sum_{k=1}^{n_m}\mathrm{M}_k\mathbf{P}_k)]\,dV \quad (21)$$

The nonlinear system of equations (12)-(13) is solved using time stepping and a Picard-Banach iteration at each time step. The convergence of the iterative procedure has been already discussed in [12], with reference to a system of bodies fixed in the space. The approach here discussed for taking into account moving bodies, due to the particular choice of the reference frames, allows for a similar treatment of the nonlinearity. In the Appendix we briefly recall the main aspects of the convergence properties of the iterative procedure. Here we recall the main steps of the formulation, from the numerical point of view.

At each time instant $t_{N+1} = t_N + \Delta t$, the average values $[B]_{k+1}^{N+1}$ of the flux density components in each element of the ferromagnetic domain, corresponding to the (k+1)-th iteration are given by the Biot-Savart law using the magnetization $[M]_k^{N+1}$, the eddy currents $[I]^{N+1}{}_k$ and external currents as:

$$[B] = \{D\}^{-1}(\{E\}[M] + \{F\}^T[I] + [W]) \quad (22)$$

From this estimate of [B] the magnetization [M] can be corrected using the numerical counterpart of the function g, leading to the following Picard-Banach scheme [16]:

$$[M] = G([B]) = G(\{D\}^{-1}(\{E\}[M] + \{F\}^T [I] + [W])) \quad (23)$$

where G is the mapping from [M] to the local average values of [B], i.e. the inverse of the operator defined by (21).

Notice that the existence of g^{-1}, which is used in (11), (13), and (21), is not implied by the existence of f^{-1}. On the other hand, equation (23), which is equivalent to (13) in case g^{-1} exists, still holds beyond saturation. We adopted g^{-1} in the weak form (11) to explicitly remark that we update **M** as the magnetization corresponding to the average value of **B** [12,16]. However, in (23), the final form of the numerical formulation, we only refer to the function g.

The nonlinear system of equations (10) and (23) is solved by time stepping and Picard-Banach iteration:

$$(\{L\}^{N+1} + \{R\}\Delta t/2)[I]_k^{N+1} = [U]^{N+1} - \{F\}^{N+1}[M]_k^{N+1} - \{R\}\sum_{i=0}^{N}[I]^i \Delta t \quad (24)$$

$$[M]_{k+1}^{N+1} = G(\{D\}^{-1}(\{E\}^{N+1}[M]_k^{N+1} + \{F\}^{N+1^T}[I]_k^{N+1} + [W]^{N+1})) \quad (25)$$

here superscript N refers to the time instant t_N, whereas subscript k refers to the k-th Picard-Banach iteration.

Motion is taken into account in (24)-(25) in the source terms [U], [W] as well as by updating parts of matrices {L}, {F} and {E} at each time step. The procedure is similar to the approach presented in [17]. For instance, the source term [U] defined by (16) is affected by the motion whenever $\mathbf{v}_k \neq \mathbf{v}_s$, because in this case the distance between the two material points $\mathbf{x} \in V_c$ and $\mathbf{x}' \in V_0$ varies.

IV. Numerical Results

Firstly, we study the problem of the moving coil over a non-ferromagnetic conducting plate presented in [18]. Two sets of results are presented for this problem for the velocity v=138.9 m/s (500 km/h). For a coarse mesh, using 1000 tetrahedra and 541 active edges, we obtained the total Joule losses $P_t = 7.1 \cdot 10^6$ W. Using a fine mesh, with 2975 tetrahedra and 1239 active edges, we obtained the total Joule losses $P_t = 8.9 \cdot 10^6$ W. These values can be compared with the analytical value indicated in [18] for an infinite conductive plate, which is $9.0 \cdot 10^6$ W, and with the corresponding numerical results $7.5 \cdot 10^6$ and $8.5 \cdot 10^6$ W [18]. In Fig. 1 we show the eddy-currents plots obtained with the fine mesh.

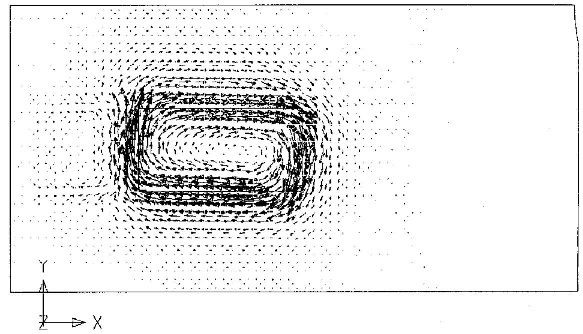

Fig. 1 The eddy-current density plot in the conducting plate for a fine mesh and for velocity of the moving coil v =138.9 m/s at t = 2.5 ms.

Results were also obtained for nonlinear magnetic media in movement. First, for testing the linear solver for moving bodies, we have considered a permanent magnet (with imposed magnetization **M** = 1 MA/m) moving near a fixed conductor non-magnetic plate, as it is represented in Fig. 2. The mesh used for **M** approximation (hexahedra) can be different from the eddy-current mesh (tetrahedra). Figure 3 shows the eddy current distribution at two different time instants, during the motion. The velocity considered is v = 1 m/s, the time-step is 1 ms.

Figure 4 shows the eddy current distribution for a ferromagnetic plate with the same geometry moving with a different velocity with respect to the magnet. In this case the velocity is 4 m/s, and the time step is 0.1 ms. Although the diffusion time is longer than in the previous case, the time step has been reduced because of the higher velocity. The maximum power is also higher because of the higher velocity and magnetic field.

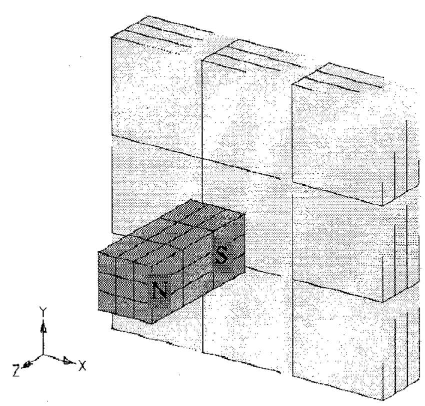

Fig. 2 Two moving bodies, a permanent magnet moving with respect to one conducting plate, considered non-magnetic, in the first example and ferromagnetic, in the second example.

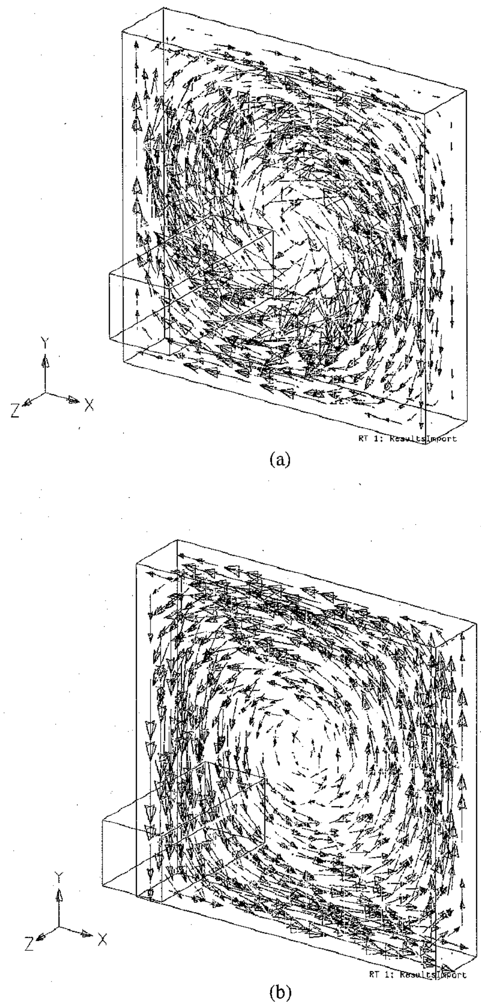

Fig. 3 The eddy-current distribution for the conducting non-magnetic plate which is moving with respect to the permanent magnet; the velocity is v= 1 m/s along z direction a) t= 1ms; b) t= 12 ms.

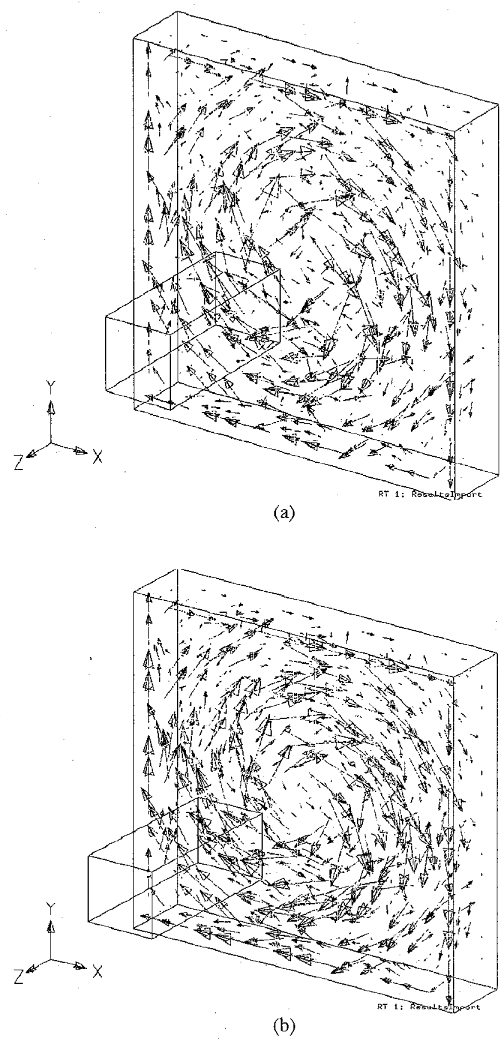

Fig. 4 The eddy-current distribution for the conducting ferromagnetic plate which is moving with respect to the permanent magnet; the velocity is v= 4 m/s along z direction a) t = 1 ms; b) t = 12 ms.

V. CONCLUSIONS

The main features of the procedure presented in the paper are here summarized.

The non-linear constitutive relation **B**-**H** (2) is replaced by a linear one (3) where the magnetization **M** is a nonlinear function of the flux density **B** given by (4). Therefore, the magnetic vector potential **A** can be calculated from **J** and **M** using the linear relationship (6).

The coordinate frames are fixed on the rigid moving bodies. As a consequence the velocity does not appear in Faraday's law (5).

In (10) the time integrals of the electric field and of the current density are considered when applying Galerkin procedure. In this way, the associated integral equation (12) is similar to that obtained for non-moving media. Motion is taken into account in the source terms and by the time-dependent behavior of the relevant matrices.

The numerical solution is carried out using an extension of the approach proposed in [7], based on a two-component electric vector potential **T** approximated via cotree edge shape functions. The magnetization vector **M** is instead approximated as a piecewise uniform field.

The matrices used for the solution of system (24)-(25) may depend on the time, but do not change throughout the Picard-Banach iteration at a time step. Therefore parallel processors can be exploited to compute to update the matrices for the next time step.

APPENDIX

Non-expansive character of the mapping from M to B

It can be shown that if the speeds of the bodies are given, if two quasi-stationary electromagnetic fields have the same boundary initial conditions, then the difference ($\Delta\mathbf{B}$,$\Delta\mathbf{H}$,$\Delta\mathbf{E}$,$\Delta\mathbf{J}$) of these fields verifies the relationship:

$$\int_0^t \int_\Omega \Delta\mathbf{B}\Delta\mathbf{H}\, dV\, d\tau = \langle \Delta\mathbf{B}, \Delta\mathbf{H} \rangle < 0 \qquad \text{(A.1)}$$

Let us assume the magnetization $\mathbf{M}$ to be defined by the following constitutive equation:

$$\mathbf{B} = \mu(\mathbf{H}+\mathbf{M}) \qquad \text{(A.2)}$$

where μ is a constant.

For a given magnetization $\mathbf{M}$ we obtain a unique flux density $\mathbf{B}$, since the function involving Biot-Savart formula $Z: L_\mu(\Omega) \rightarrow L_\nu(\Omega)$ is well defined, where $\mathbf{B}=Z(\mathbf{M})$ and $\langle \mathbf{X}, \mathbf{Y} \rangle_\mu = \langle \mathbf{X}, \mu\mathbf{Y} \rangle$. From (A.1)-(A.2) we get:

$$\|\Delta\mathbf{B}\|_\nu^2 \le \langle \Delta\mathbf{B}, \Delta\mathbf{M} \rangle \le \|\Delta\mathbf{B}\|_\nu \|\Delta\mathbf{M}\|_\mu$$

Therefore function Z is non-expansive:

$$\|Z(\mathbf{M}') - Z(\mathbf{M}'')\|_\nu \le \|\mathbf{M}'-\mathbf{M}''\|_\mu \qquad \text{(A.3)}$$

Contractive character of the mapping from B to M

We suppose that the constitutive relation (2) verifies Lipschitz condition:

$$\|f(\mathbf{B}') - f(\mathbf{B}'')\| \le \Lambda \|\mathbf{B}' - \mathbf{B}''\|, \quad \forall \mathbf{B}', \mathbf{B}'' \in L^2(\Omega) \qquad \text{(A.4)}$$

and is uniformly monotone:

$$\langle \mathbf{B}'-\mathbf{B}'', f(\mathbf{B}') - f(\mathbf{B}'') \rangle \ge \lambda \|\mathbf{B}'-\mathbf{B}''\|^2, \ \forall \mathbf{B}', \mathbf{B}'' \in L^2(\Omega) \qquad \text{(A.5)}$$

where $\Lambda \ge \lambda > 0$. We may choose ν so that the function g defined by $g(\mathbf{B}) = \nu\mathbf{B} - f(\mathbf{B})$ is a contraction:

$$\|g(\mathbf{B}') - g(\mathbf{B}'')\|_\mu < \theta \|\mathbf{B}'-\mathbf{B}''\|_\nu, \ \forall \mathbf{B}', \mathbf{B}'' \in L_\nu^2(\Omega) \qquad \text{(A.6)}$$

where $\theta < 1$. Indeed, if we choose $\mu = 1/\nu \in (0, 2\lambda/\Lambda)$, then the contraction factor θ of the function g is:

$$\theta = \frac{\|g(\mathbf{B}') - g(\mathbf{B}'')\|^2}{\|\mathbf{B}' - \mathbf{B}''\|^2}$$

$$= \frac{\|\mathbf{B}'-\mathbf{B}''\|^2 - 2\mu < \mathbf{B}'-\mathbf{B}'', f(\mathbf{B}') - f(\mathbf{B}'') > + \mu^2 \|f(\mathbf{B}') - f(\mathbf{B}'')\|^2}{\|\mathbf{B}' - \mathbf{B}''\|^2}$$

$$\le 1 - 2\mu\Lambda + \mu^2\Lambda^2 < 1 \qquad \text{(A.7)}$$

Convenient criteria for the choice of the permeability μ are given in [16]. There is proof that $\mu = \mu_0$ may be used and in this case we obtain $\theta = 1 - \mu_0/\mu_{max}$, where μ_{max} is the maximum value of the differential permeability in Ω.

Convergence of the Picard-Banach procedure

The method is a Picard-Banach iteration yielding the fixed-point of the contractive function $g \circ Z$.

It is possible to obtain a posteriori error estimates. There is the following upper bound for the error in comparison with the exact solution:

$$\|\mathbf{B} - \mathbf{B}_k\|_\nu \le \|\mathbf{M}_{k+1} - \mathbf{M}_k\|_\mu / (1-\theta) \qquad \text{(A.8)}$$

Convergence of the numerical approach

In the numerical formulation a number of additional approximations are introduced. The current density and the magnetization vector are approximated by (8) and (9), the Galerkin approach (10)-(11) is used to impose the constitutive equations, time-stepping is employed for the integration of (12), and the Picard-Banach iteration for the nonlinear system at each time step is carried out by (24)-(25).

Here we focus on the convergence of the Picard-Banach iteration used in the numerical approach.

The numerical approximation of the linear mapping Z used in the iterative procedure is given by:

$$[\mathrm{M}]_{k+1}^{N+1} = G([\mathrm{B}]_{k+1}^{N+1}) \qquad \text{(A.9)}$$

$$[\mathrm{B}]_{k+1}^{N+1} = \{T\}[\mathrm{M}]_k^{N+1} + [S] \qquad \text{(A.10)}$$

where $\mu = \mu_0$, $\{T\} = \{D\}^{-1}(\{E\}^{N+1} - \{F\}^{N+1}\{A\}^{-1}\{F\}^{T\ N+1})$, $\{A\} = \{L\}^{N+1} + \{R\}\Delta t/2$, and $[S]$ is a term that does not depend on $[\mathrm{M}]_k^{N+1}$.

The contractive character of (A.9) follows from (A.6) and the non-expansive character of the average operation [16]. The non-expansive character of (A.10) follows from the analysis of the eigenvalues of the iteration matrix {T}, which are all real and less than μ_0 [13].

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