Eddy Current Solver for Nondestructive Testing using an Integral-FEM Approach and Zero-Thickness Flaw Model

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An Integral-FEM approach is proposed for solution of eddy-current problems in Nondestructive testing. Simulation of zero-thickness 2D cracks is possible and large datasets can be simulated with an overall reduced computational effort.

***Index Terms*—Integral-FEM, ECT, NDT.**

I. Introduction

Several methods were proposed for fast and accurate solution of eddy-currents in conductive media in view of inversion of probe signals for extraction of defect geometry in nondestructive testing: both FEM-BEM [1][2] or FEM [3] based. One significant limitation of all these methods appears when one need to model defects with very thin geometry, to simulate the natural cracks. Zero-thickness cracks may be approximated using volume elements with very small dimension and in this case mesh refinement will increase very much the problem size, the solution being therefore unpractical for calculation of large datasets required for signal inversion. Very thin crack geometry, approximated as zero-thickness cracks are treated in [4]. Zero-thickness cracks are also modeled using a formulation based on the circulation of magnetic vector potential along primal edges and on time-integrated electric scalar potential in primal nodes of conducting region [5].

II. Formulation

The proposed method is based on application of **T**- electric potential to the integral equation of eddy currents, like in [4] and [6][7]. Starting from Maxwell equations in quasi-stationary form and the constitutive relationship:

$$\boldsymbol{E} = \rho \cdot \boldsymbol{J} \,, \tag{1}$$

where $\boldsymbol{J}$ is the current density, $\boldsymbol{E}$ is the electrical field and ρ is the resistivity in the conductive domain Ω_c. We suppose that the field sources motion relative to the conductive domain is slow and therefore the component of the induced field through motion is very small and negligible. In the laboratory frame, the electrical field is:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla V \,, \tag{2}$$

where V is the electric scalar potential and $\boldsymbol{A}$ is magnetic vector potential. The magnetic vector potential can be calculated using Biot-Savart formula:

$$\mathbf{A} = \frac{\mu_0}{4\pi}\int_{\Omega}\frac{\mathbf{J}}{r}\,\mathrm{d}V + \mathbf{A}_0 \tag{3}$$

where $\boldsymbol{A}_0$ is the magnetic vector potential due to the impressed current sources:

$$\mathbf{A}_0 = \frac{\mu_0}{4\pi}\int_{\Omega_0}\frac{\mathbf{J}_0}{r}\,\mathrm{d}V \,, \tag{4}$$

and Ω_0 is the air. Only conductive media are meshed. The current density is expressed in terms of shape functions associated to the edges in the inner co-tree [6]:

$$\boldsymbol{J} = \sum_{k=1}^{n} \boldsymbol{N}_k \nabla \times \boldsymbol{T}_k \,. \tag{5}$$

Applying Galerkin approach, the following equation system is obtained:

$$[R][I] + [L]\frac{\mathrm{d}[I]}{\mathrm{d}t} = [U] \,. \tag{6}$$

where the terms of matrices $[\boldsymbol{R}]$ and $[\boldsymbol{L}]$ are calculated as:

$$L_{ij} = \frac{\mu_0}{4\pi}\int_{\Omega_c}\int_{\Omega_c}\frac{\nabla\times\mathbf{T}_i \cdot \nabla\times\mathbf{T}_j}{r}\,\mathrm{d}v_c\,\mathrm{d}v_c \,; \tag{7}$$

and respectively:

$$R_{ij} = \int_{\Omega_c}\nabla\times\mathbf{T}_i \cdot \rho\nabla\times\mathbf{T}_j\,\mathrm{d}v_c \,. \tag{8}$$

and the right-hand side term U_i is calculated as:

$$U_i = -\frac{\partial}{\partial t}\left(\frac{\mu_0}{4\pi}\int_{\Omega_c}\int_{\Omega_0}\frac{\nabla\times\mathbf{T}_i \cdot \mathbf{J}_0}{r}\,\mathrm{d}v_c\,\mathrm{d}v_0\right) \tag{9}$$

with Ω_0 being the domain of impressed currents and $\mathbf{J}_0$ being the current density inside Ω_0. U_i results from $\mathbf{A}_0$ component of

A, projected on the shape functions and integrated over the whole conductive domain Ω_c.

In order to model 2D, zero-thickness defects, from the set of inner co-tree edges are eliminated those edges placed in the defect surface. The procedure is equivalent to zeroing the circulation of scalar electric potential **T** on those co-tree edges. For harmonic excitation, the system can be treated in AC regime. Although the system matrix is full, longitudinal scans (B-scans) with slow motion of an AC-excited probe can be simulated in a multi-step AC simulation with only one system matrix inversion. Applying a database approach, calculation of a multitude of defect geometries can be done with significantly reduced CPU effort, in comparison with classical FEM-scheme.

Crack geometries can be represented in two ways. One approach is to simulate cracks with finite, non-zero thickness, like most FEM and FEM-BEM solvers. Another approach is to simulate zero-thickness crack surface by vanishing the circulations of **T** in all co-tree edges in the defect surface. For reconstruction of real cracks, the second method may prove to be more accurate.

III. Numerical Results

In order to validate the numerical method proposed, we start by validating the formulation without the 2D crack model; for this purpose, we first show results for TEAM Workshop benchmark problem no. 15 (Rectangular slot in a thick plate: A problem in nondestructive evaluation) [8], for the first frequency, of 900 Hz. The problem setup, plate material and coil dimensions and excitations are detailed in [8]. Fig. 1 shows comparison of the experimental results presented in [8] with the simulated results by proposed method.

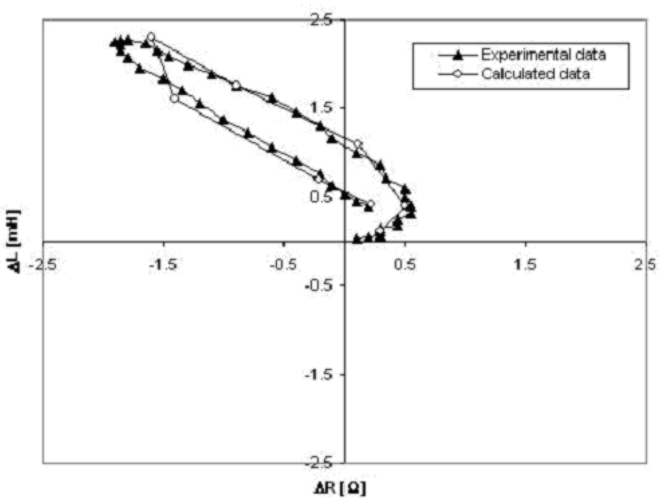

Fig. 1. Coil impedance; TEAM Workshop problem 15, f = 900 Hz.

Although the mesh is not very fine, the agreement is acceptable.

We apply the zero-thickness (2D planar surface) model to a second problem, an Inconel 600 tube excited with a pancake-type coil, placed inside the tube. The problem definition is given in Table 1. Several tests are performed, using several values for exciting coil lift-off, flaw length and flaw depth.

TABLE I
Parameters for test problem: Stainless stell - Inconel 600 tube excited by a pancake shaped coil

	Parameter	Value
Pancake coil	Inner radius (a2)	0.9 mm
	Outer radius (a1)	1.5 mm
	Length (b)	0.6 mm
	Turns (N)	140
	Lift-off	• 0.5 mm • 0.2 mm
Test specimen	Conductivity (σ)	1.0×10^8 S/m
	Thickness	1.27 mm
	Inner radius	10.5 mm
Flaw	Length (2c)	• 4 mm • 2 mm
	Depth (h)	• 0.635 mm (50% ID) • 0.3175 mm (25% ID) • 0.9525 mm (75% ID)
	Width (w)	Zero-thickness (2D crack)
Other	Frequency	10 kHz

Only inner defects (ID) are treated. A mesh with 4440 elements and 3705 active edges (in the case with no crack) was used for the simulations. For each case, both the case with the defect and the case without the defect are analyzed and coil impedance is calculated. The difference signal is then plotted. Fig. 2 shows the problem mesh for all calculation cases for test problem described in Table 1.

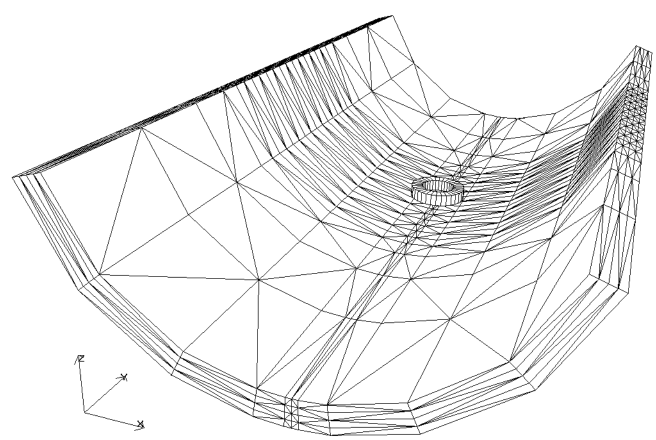

Fig. 2. Problem setup: tube mesh and excitation coil. Mesh with 4440 elements and 3705 active edges (edges in inner co-tree).

In all test cases we model 2D surface – zero-thickness cracks oriented longitudinal, along the tube axis (in y direction, in Fig. 2). The scan starts from the center position over the crack and with a 0.5 longitudinal scan step pitch. Fig. 3 plots the difference signal of complex coil impedance (induction vs. resistance), for 2 cases: 2 mm length crack and 4 mm length crack, for depths of 0.635 mm (50% ID) and lift-off 0.5 mm.

In Fig. 4 we show the results for the same difference signal and for defect depths: 25%, 50% and 75%, for the 4 mm long defect and with excitation coil with a 0.5 mm liftoff.

Fig. 5 presents the comparison between cases of longitudinal scan with excitation coil having a 0.2 and 0.5 lift-off. The defect has 4 mm length and is 50% in both cases.

For all these simulations, 10 steps scans were calculated. All steps are calculated in the same time, the system matrix being inverted only once.

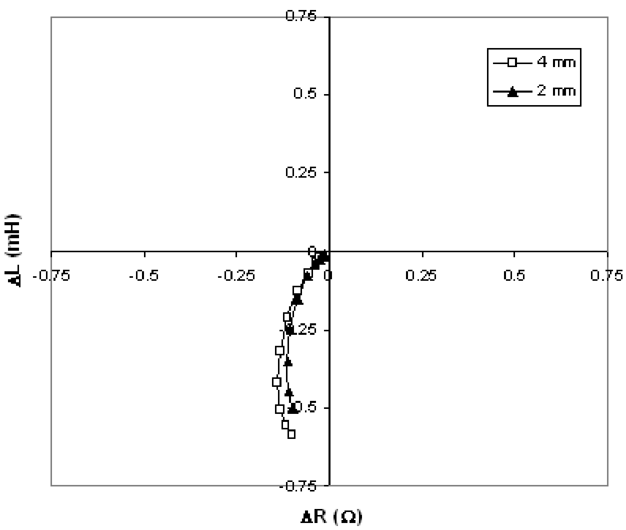

Fig. 3. Inner defect 0.635 mm (50% ID), crack length 2 and 4 mm, oriented along tube axis. Longitudinal scan.

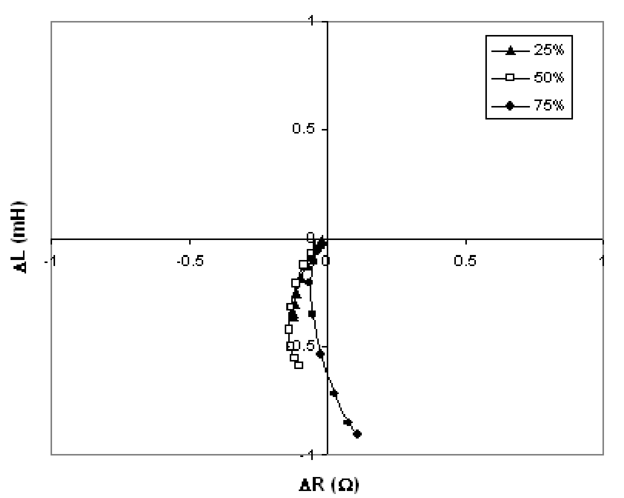

Fig. 4. Inner defect, length 4 mm, oriented along tube axis. Scan is longitudinal. Comparison between 3 cases of defect depth: 25%, 50%, 75% ID cracks.

IV. Conclusion

We presented a method for simulation of eddy-currents problems with application at non-destructive testing. The proposed method has two strong points, when applied as the forward solver for inversion of simulated signals in eddy current testing (ECT): (1) zero-thickness cracks can be simulated without approximation, by zeroing the circulation of electric vector potential in the surfaces that define the crack, modeling thus with better the natural crack problems and (2) large databases of signals can be calculated with limited CPU time, because of mesh size reduction. Therefore application of signal inversion methods for defect geometry and position reconstruction becomes affordable.

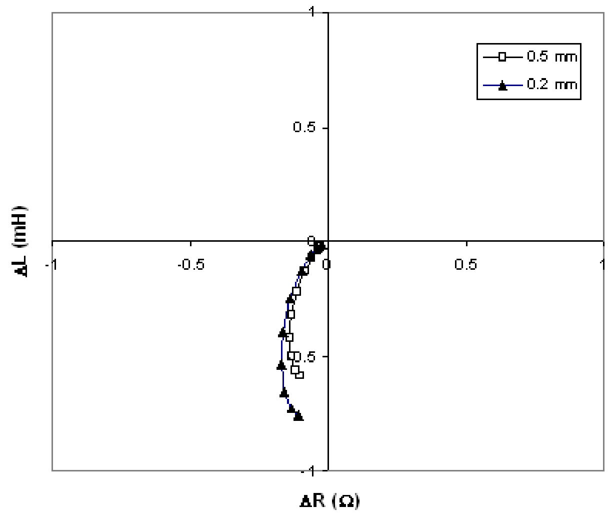

Fig. 5. Inner defect 4 mm length, 50% inner defect, longitudinal scan. Comparison between cases of 0.2 mm and 0.5 mm lift-off for the excitation coil

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